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THE MATHEMATICS TEACHER

Volume XXXI Number 1

Edited by William David Reeve

The Enrichment of the Mathematics Course*

By H. E. RITER
Provincial Normal School, Winnipeg, Manitoba, Canada

THE growing desire among teachers of mathematics to make their subject significant in the lives of all their pupils has led them to seek out means of enriching the course beyond the narrow confines of text book material or prescribed courses of study. To this end they have attempted to link up mathematics with real life situations where it is used, to acquaint pupils with something of its long history and social significance, and to develop an appreciation and interest that will lead to voluntary investigation and study. With the lessening pressure of outside examinations in our own high schools it would appear that there is a present opportunity for worth while achievement along this line.

It is the purpose of this paper to suggest some ways in which this enrichment of the mathematics course may be attempted. First let us consider the possibilities for enrichment that lie in the history of mathematics. History teaches us that man learned to count only after a long and painful struggle, lasting from the dawn of intelligence down to comparatively modern times. In this struggle he was driven by the necessities of his life and handicapped by the awkwardness of the systems of notation developed from time to time.

The following questions and suggestions are offered as an example of the kind of historical material that might appeal to elementary pupils.

1. Following a discussion of the lives of primitive peoples raise the question of how they could express number without number words—suggest the use of fingers, pebbles, twigs, notches on a stick, knots in a cord, marks in the sand.

2. Why do we think and work with our numbers expressed in groups of ten? Was it because our ancestors counted with their ten fingers?

3. What does 23 mean? Two tens and 3. What would it likely mean if man had been created with 8 fingers instead of 10? Would it be 2 eights and 3, our nineteen?

4. For older pupils a little work in a scale other than ten is interesting and illuminating.

5. Why do you suppose the Zulu calls the number six, 'taking the thumb,' and the number seven, 'he pointed'?

6. Why do the Indians of the Orinoco River call eleven 'one down'; twenty, 'one Indian'; twenty-one, 'one on the other Indian'?

Of course 'one down' meant down on his toes.

7. The awkwardness and primitive form of the Roman system as compared with our own system of writing numbers can be easily shown. Let the pupils try calculating with it. Of course the Romans had more sense than to try. They used it for writing numbers only and calculated with counting frames.

8. The wooden rectangular frame having wires with moveable beads strung thereon, found in many primary rooms, is an example of the counting frames used for calculating for

many centuries before the introduction of the Hindu-Arabic notation. The pupils will be interested to hear that the children of China and Japan are still taught its use in their schools today. They might try expressing numbers with it by naming the wires in order units, tens, hundreds, etc. Addition and subtraction can be easily done with it but division is hard, being done by repeated subtraction.

9. The invention of the zero by the Hindus and its use to mark blank columns, as in 300, made it possible to express numbers without the counting frame and to calculate with symbols rather than counters. This seemed a marvellous invention to the people of Europe when it was brought by the Moors or Arabs to Spain about 1000 A.D. It was so easy to calculate with the new notation and the zero that after a few hundred years the counting frames were discarded altogether, though not without opposition from people who did not understand the system.

Possibly enough has been said to suggest the uses of the history of mathematics for enriching the course in arithmetic for younger pupils. For older pupils who are studying geometry, even in grade seven, something of the achievements of the Egyptians and Greeks might be given.

The Egyptian engineers knew enough of mathematics to enable them to survey their fields, to build great temples and pyramids that are still the wonder of the world, and to construct an excellent calendar 4241 years before the birth of Christ. In the British Museum today there is a book on mathematics written by a priest Ahmes, about 1500 B.C. in which he refers to an older book probably written about 2300 B.C. Some of his rules were wrong, for example, the area of an isosceles triangle was given as $\frac{1}{2} \times$base$\times$one side and the area of a circle as $8/9 \times$diameter squared. The class might explain what is wrong. The wonder is that so much was discovered and used so long ago.

If it is as important that pupils should know the great conquerors in the field of human thought as that they should know the conquerors in the field of battle, then some of the great Greek mathematicians should be studied.

1. They should hear of Thales, that Greek merchant of enquiring mind who traded in Egypt and was so amazed at the temples, pyramids, and engineering works of that land, ancient even in his day, that he stayed to study the mathematics which made these things possible. He learned from the Egyptian priests and astonished his teachers by calculating the height of a pyramid from the length of its shadow as compared with that of a stick upright in the sand. Pupils will be interested in trying out his method of shadow reckoning and they may easily be led from this to a consideration of similar triangles and tangent ratios.

The angle-side-angle case in congruent triangles takes on added significance if the class is told how Thales used it to find the distance of a ship off shore. He sighted the ship from a tall tower and then sighted, at the same angle, an object along the beach. The two congruent right angled triangles will be clear in a diagram.

2. The class will be interested to hear something of Pythagoras when studying his famous theorem concerning the sides of a right angled triangle. Mention might be made of his secret society and their mathematical discoveries, of their symbol, the five point star, and its construction. A correlation with physics and algebra will be found in the discovery of Pythagoras that strings of lengths 6, 4, & 3 give, respectively, a note, its fifth, and its octave. He called 6, 4, & 3 a Harmonic progression, and noted that their inverses $\frac{1}{6}$, $\frac{1}{4}$, and $\frac{1}{3}$ were in Arithmetic Progression i.e. $\frac{1}{4} - \frac{1}{6} = \frac{1}{3} - \frac{1}{4}$.

3. Plato, the world's greatest philosopher, thought so much of geometry as an intellectual exercise and a way to truth, that he had inscribed over the door of his academy at Athens, "Let no one ignorant of Geometry enter here."

4. Euclid organized the geometric knowledge of his day into a book so perfect in its logic that it has been used throughout the civilized world for 2300 years as a model of accurate reasoning.

5. It is a startling and significant fact that 1700 years before Columbus was ridiculed for thinking the earth a sphere, Eratosthenes, a professor in the University of Alexandria, succeeded in measuring its circumference to within 50 miles.

6. Archimedes was successful as an inventor and discoverer because he was able to apply mathematics to practical matters.

Perhaps the most important principle to be learned from the history of algebra is that symbols and an efficient notation help man to think. The development of modern algebraic symbolism led to the invention of new kinds of number that were developed theoretically by mathematicians long before practical uses for them were found.

For example, $x-1=0$ led naturally to a consideration of the meaning of $x+1=0$ which was explained by the invention of negative number $x=-1$.

$x^2-2=0$ led to the invention of irrationals, $x=\sqrt{2}$.

$x^2+1=0$ led to the system of imaginaries, $x=\sqrt{-1}$, the application of which to the measurement of alternating currents made possible the control of electricity in the service of man.

The very names of these numbers, negative, irrational, imaginary, suggest the early attitude towards the seemingly useless imaginings of mathematicians.

Our word algebra comes from the Arabic "al-jabr" which means 'restoration.' An Arab, al-Khowarizmi, used the word to mean about the same as our term transposition, in 825 A.D. The Moors taught the subject in Spain where the word "algebrista" still means bone *restorer*, or as we should say, 'bone setter.'

Projects and field work offer much in the way of enrichment. This takes time, but for many pupils it will mean the difference between working blindly by rule and working with full comprehension of the meaning and purpose of mathematics.

In the lower grades we may use:

1. Maps and scale drawings of school buildings and grounds.
2. Floor plan, side, and front elevations of a simple granary or garage, with calculation of probable material and labor costs.
3. Dramatization by pupils of such transactions as banking, insurance, commission, taxation, etc.
4. Carrying a straight line over the ground and measuring it. Lining up stakes or poles gives the line. Fencing ,and plowing the first furrow to strike out a land, are common applications.
5. Measuring angles in the horizontal plane may be done with a flat board on which is drawn a semicircle with radii at intervals of say 5 degrees. The board may be levelled with a spirit level or with the old Egyptian level, a wooden isosceles triangle with a plumb line dropped from the vertex to bisect the base. Long pins may be used for sighting.

Older pupils may try:

1. Mapping—measure a base line on the ground and the angular direction of an object as sighted from each end of this line. Now draw the base line to scale on paper. When the direction lines to the object are then drawn they will intersect at the position of the object on the map. A whole district may be mapped in this way using one base line.
2. Measuring an angle in the vertical plane —clamp a large blackboard protractor to an upright stick. Check the vertical position of the stick by a plumb line and sight with one pin at the center of the protractor and others at the circumference. Measuring heights and distances —measure angle of elevation of a tall object and distance to its base. Find height by scale drawing, similar right angled triangles, or tangent table according to grade level of pupils.

Find latitude by measuring elevation angle of North Star.

Find north south line, the meridian, by setting a stick upright at center of a circle on the ground and noting two points at which the tip of the moving shadow lies just on the circle. Bisect the arc between these points and join bisection point to the stick. This is the meridian line.

The organization of Mathematics Clubs is also suggested as a stimulus to mathematical study and an instrument of enrichment. Membership should be confined to those interested and be voluntary. Regular meetings may be held in school time. Committees may report on special topics, outside speakers may be had on occasion, and the club may sometimes entertain the whole school with a display, lecture, or mathematical play. (See the *Mathematics Teacher* for plays of this kind, 525 West 120th St., New York.)

The following topics might serve for study and report in the clubs:

Historic Notations
Ancient Calculating Devices
The History of Our Calendar
The History of Our Common Measures
Great Mathematicians
Mathematics in Navigation of Sea and Air
Famous Problems
Mathematical Puzzles
Modern Calculating Machines
Mathematical Fallacies
Magic Squares

See these articles in THE MATHEMATICS

TEACHER:

Jan. 1930—by Margaret Steward
Feb. 1930—by Byron Bentley
Feb. 1933—by Norman Anning

Mathematical recreations may be found in such books as "Fun With Figures" by Collins, published by Appleton, and "Boys Own Arithmetic" by Weeks, published by E. P. Dutton & Co.

Finally pupils should be led to appreciate the progressive, dynamic nature of mathematical science today. While they can hardly appreciate the nature of modern mathematical research, they can at least see how its applications grow with world needs. They can see decimals replacing common fractions, the French metric system displacing other national systems, and the growing use of graphs in the daily press.

The use of pictorial material is especially stimulating. The following materials are submitted as suggestive.

1. Tree of Knowledge—a large poster showing mathematics as the basic science. Price 25¢—Address, Business Manager, Museum of Science & Industry, Jackson Park, Chicago.

2. Six pictures showing the development of common measures. Price $2 for set in plain finish. Address H. G. Ayre, State Teachers College, Macomb, Ill.

3. Three murals on the development of mathematics. Price 90¢ set (old quotation). Address, Bureau of Publications, Teachers College, University of Columbia, New York.

4. Historical Time Chart of Mathematics, by E. J. Edwards; University of London Press. Set of 5, one guinea.

Among useful books may be mentioned:

"History of Mathematics" by D. E. Smith, 2 volumes; Ginn & Co.

"History of Mathematics" by Sanford; Houghton Mifflin.

"A Short Account of the History of Mathematics" by Ball; Macmillan Co. "Numbers and Numerals" by Smith & Ginsberg; Bureau of Publications, Teachers College, Columbia University, New York. "Mathematics for the Million" by Hogben; Geo. Allen & Unwin Ltd, London, or W. W. Norton and Co., New York.

SALE ON YEARBOOKS

We have been fortunate in securing a few more copies of the *First Yearbook* which is now out of print. We can, therefore, offer a complete set of yearbooks (numbers 1 to 12 inclusive) postpaid for $16.50, a saving of 20%.

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PLAYS

Back numbers of *The Mathematics Teacher* containing the following plays may be had from the office of *The Mathematics Teacher*, 525 West 120th Street, New York.

A Problem Play. Dena Cohen.
Alice in Dozenland. Wilhelmina Pitcher.
An Idea That Paid. Florence Brooks Miller.
Mathematical Nightmare. Josephine Skerrett.
Mathesis. Ella Brownell.
The Eternal Triangle. Gerald Raftery.
The Mathematics Club Meets. Wilimina Everett Pitcher.
The Case of "Matthews Mattix." Alice K. Smith.
More Than One Mystery. Celia Russell.
Out of the Past. Florence Brooks Miller.

Price: 25¢ each.

Everyman's Visit to the Land of the Mathematicians

An Imaginary Journey to the Abode of Some of the Great Mathematicians

By Edith Bruce Paterson
Baltimore, Maryland

Introduction

This sketch of the history of mathematics is intended for a junior high school audience, although it would not be too simple for a senior high school audience.

The scene is laid in an imaginary land where the great mathematicians of the past have congregated after their lines on earth were ended. Therefore we find men widely separated in time speaking to each other. The organization of the episodes into subjects and the conversations was chosen in order to bring out the abstract character of pure mathematics, and by reference to applications of mathematics to show that purely theoretical work usually, though not always, preceded the practical applications. The closing paragraph of the epilogue states the purpose of the play.

The following authors and publishers have kindly granted permission for the reproduction of pictures and quotations:

Mr. Claude Bragdon and his publishers, Alfred A. Knopf, Inc.; for the two illustrations: "Sinbad, lost in the desert, discovers the five Platonic solids," from *The Frozen Fountain;* and "The hexagram and equilateral triangle in nature" from *The Beautiful Necessity.*

Ginn and Company for the quotation of the quatrain found on page 389 of the second volume of David Eugene Smith's History of Mathematics.

Professor H. W. Turnbull and his publishers, Methuen and Company, Ltd. for the quotation of the last paragraph of *The Great Mathematicians.*

Production Notes

Scenery: The backdrop was an adaptation of Claude Bragdon's drawing as indicated in the tracing. Blue cardboard (the color is not important) was cut into strips about five inches wide and glued into the figures, then pinned to the curtain at the back of the stage.

In Episode III kindergarten tables covered with artificial grass mats were used. The same scene was used in Episode IV. No scenery other than the backdrop was used in Episodes I, II, and the interlude.

In Episode V two period chairs, a period table, and a large globe were grouped in the center of the stage.

Posters: "The hand of God" from the Riccioli frontispiece was hung on the proscenium. The hexagram poster and the equilateral triangle poster were placed on easels at the side of the stage where they remained throughout the performance. Vitruvius carried two posters, one of the restoration of the Parthenon and one of the Sphinx and Pyramids. All of the posters were drawn boldly on tagboard with india ink.

Properties: A rope knotted so as to form the 3-4-5 triangle was carried by the Egyptians, and a bamboo pole borrowed from the gymnasium was used to raise it aloft.

One small scroll is needed for the Greeks.

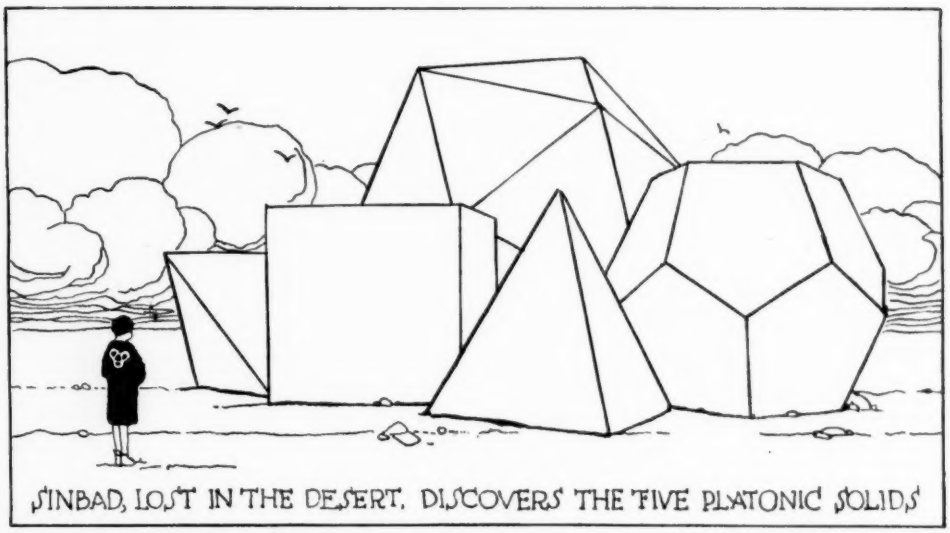
Grosseteste carried a large folded piece of paper, sealing wax prominently displayed.

A reproduction of the Roman abacus was carried by the Roman.

Two old books with leather bindings were used by those who had to have books.

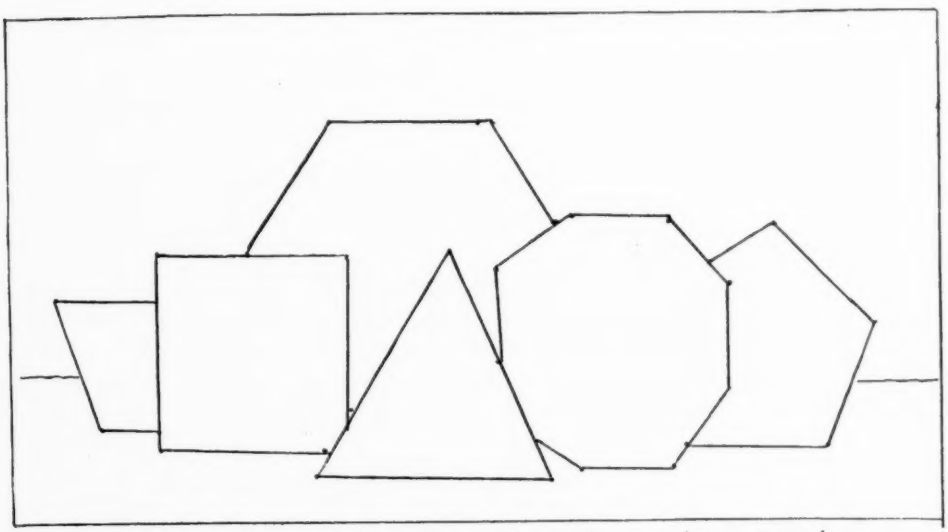
Costumes: Everyman wore black trousers, a dark blue jacket, and a red beret.

The Egyptians had headdresses made

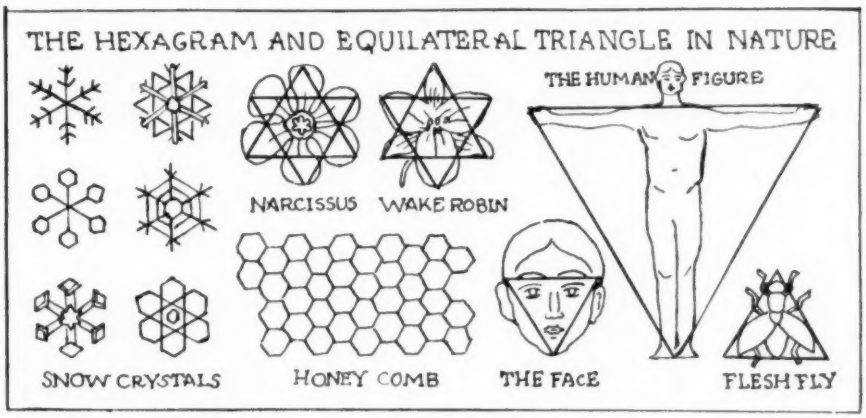


SINBAD, LOST IN THE DESERT, DISCOVERS THE FIVE PLATONIC SOLIDS

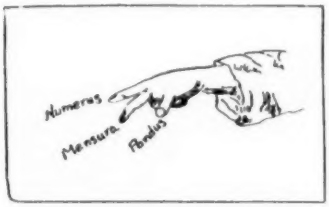
From Bragdon, Claude, *The Frozen Fountain*, N.Y. Alfred A. Knopf, 1932
Reproduced by permission of Mr. Bragdon and Alfred A. Knopf, Inc.



The adaptation of Mr. Bragdon's picture above made of strips of cardboard about five inches wide and pinned to the curtains at the back of the stage.



From Bragdon, Claude, *The Beautiful Necessity*, N.Y., Alfred A. Knopf, 1922. Reproduced by permission of Mr. Bragdon and Alfred A. Knopf, Inc.
(This was divided into two posters, the hexagrams on one, and the equilateral triangles on the other.)



From the reduced facsimile of the frontispiece in Riccioli: *Almagestum Novum*, Bologna, 1651 as reproduced in Stimson, Dorothy, *The Gradual Acceptance of the Copernican Theory of the Universe*, New York, 1917

of turkish towels folded like Red Cross headdresses. They wore white shorts and had turkish towels wrapped around the upper parts of their bodies as far as their arm-pits.

The Greeks and Romans wore Roman togas.

As far as it was possible to do so, the following characters wore costumes in keeping with their periods: Stevin, Vieta, Cardan, Recorde, Napier, Descartes, Newton, and Leibniz.

The Arab wore an Arabian costume.

Grosseteste wore a college gown with a hood, and a cap of velvet such as is worn by some church choirs.

Fibonacci wore a college gown faced with red material, and a headdress of red draped to resemble his cap in his portrait.

Prologue

(*Spoken by* Everyman *before the curtain.*) My name is Everyman. I represent each one of you. Everyone, from the youngest child in the first grade to the oldest among you is taught mathematics. Few of us, however, realize the vastness of the subject because it is difficult for those who have not gone far beyond arithmetic to appreciate its magnitude.

This is a mathematical world we live in. Nature itself seems to be governed by

mathematical laws. We see geometric shapes in flowers, plants, trees, crystals, —indeed Plato is said to have remarked: "God eternally geometrizes." Animals and insects show evidence of an innate number sense. Bees, for example, illustrate their mathematical instinct by using the hexagon in building their honey combs.

Savage man found that he needed to count and measure early in his existence on the earth. From these humble beginnings, Mathematics, the Queen of the Sciences, slowly developed. The all-pervasiveness of mathematics was realized early. This picture was taken from a book published in 1651 and it represents pictorially words from a Hebrew book written in the first century B.C. It shows the hand of God, one finger marked "number" the second "measure," and the third "weight." It refers to these words: "But thou hast ordered all things in measure, number, and weight."

Much of the mathematics we study today, however, was not inspired by the deliberate attempt to create something useful. The theories and processes were created by mathematicians following a planned and definite line of reasoning, much as a game is played according to certain rules. Jacobi, a famous mathematician, said that the true end of mathematics was the greater glory of the human mind.

Keep that statement in mind—try to forget for a while that man frequently expects some material return for his labor. The great men of the world are content to work for the work's sake with no thought of material reward. Realize that mathematics shows as perhaps no other study does the power of man's mind. Learn how wonderful were the achievements of the men who made mathematics and laid the foundations for our marvellous mechanical civilization—how much more wonderful than the machines themselves were the minds that made them possible.

I am, as your representative, going to meet some of these great men and hear from them the story of their work. I shall have the opportunity to meet only a few of them, because there are so many. Use your imaginations, pretend that we are in a land where all the great mathematicians of the past have congregated, and try to catch from them some glimpse of the true grandeur of mathematics.

GEOMETRY

Episode I

(*Curtain parts, disclosing stage dimly lit.* EVERYMAN *is seen at left, looking at the background. He turns and speaks.*)

EVERYMAN: This must be the place where the mathematicians have gathered.

(PLATO *enters from the right.* EVERYMAN *meets him at center.*)

EVERYMAN: Sir, am I in the land of the mathematicians?

PLATO: You are. Who are you, and why have you come here?

EVERYMAN: I am Everyman. I have been taught arithmetic for six years and I have heard that there is much more to mathematics than mere arithmetic. I wanted to find out, that is why I am here. Can you tell me? Are you one of the great mathematicians?

PLATO: "Wanted to find out"—that is what impels men to make discoveries in any field. I am Plato and I lived in Greece about two thousand, four hundred years ago, about 430 B.C. I am not one of the great mathematicians, but I am here because I knew how important mathematics was and insisted that young people should be taught mathematics. I had a school, very much like your colleges, and I had these words put over the door: "Let no one ignorant of geometry enter here."

EVERYMAN: Geometry—who discovered it?

PLATO: The word *geometry* comes from the Greek words *geo* meaning earth and *metron* to measure. Geometry means *earth-measure*. We Greeks got our first ideas of geometry from the Egyptians who used it in measuring land.

(TWO EGYPTIANS *bearing knotted rope*

enter left and proceed to stake out triangle. They stop work only to make speeches, then resume it again. They move towards the right.)

PLATO: Here are two of them, we shall see how they worked.

(PLATO *and* EVERYMAN *advance towards* EGYPTIANS.)

Tell this youth who you are and what you are doing.

FIRST EGYPTIAN: We are surveyors, attached to Pharaoh's tax department. Every year when the Nile floods the valley all boundaries are washed away and we must put them back so that each man may know his now land again.

SECOND EGYPTIAN: We are using this rope with these knots—we fasten it here—and here—and here.

(*Addressing* PLATO): Hold this, sir, so that we may raise it.

(PLATO *slips loop over his staff and raises it, so that rope is taut; the* EGYPTIANS *hold knots on floor.*)

Now we can use this to relocate the boundaries.

EVERYMAN: A right triangle! How did they do that? If I tied two knots in a rope and stretched it so, would it always make a right triangle?

(PLATO *lowers staff. The* EGYPTIANS *proceed staking towards right and exit when they reach door.*)

PLATO: Not unless you tied the knots in particular places. These men are called harpodonaptae—rope stretchers. Pythagoras, one of the Greek mathematicians, is said to have proved the principle that underlies the Egyptian rope-stretching.

EVERYMAN: These Egyptians must have known a great deal of geometry.

PLATO: Yes, they knew many practical applications, they used them in building their Pyramids and their huge temples; but they did not know why these practical applications always worked. The Greeks studied that—and there is the man who began that study.

(*Enter* THALES *from right, gazing at the stars as he walks slowly. He bumps into* PLATO, *as* PLATO *and* EVERYMAN *approach him.*)

PLATO: Look out, Thales, old fellow, bring your eyes down to earth or you'll be in a ditch again.

THALES (*with difficulty, brings himself to earth again*): Oh! it's you, Plato. Who's that with you?

PLATO: This is Everyman. He is visiting us to find out about mathematics. Tell him something about yourself.

THALES: Now isn't that a nice thing to ask a man to do—talk about himself! What do you want me to tell him—how I predicted the eclipse of the sun in 585 B.C. or how I developed abstract proofs about lines, or how I cornered the olive market—or what?

PLATO: That's enough. Without going into detail, he knows that you are an astronomer, a geometrician, and a shrewd business man. And, Everyman, he did excellent work in all of those fields.

(THALES *has turned and resumed his star-gazing. He moves slowly to the right during the next speech and goes out when he reaches the door. Meanwhile* PYTHAGORAS *enters from left in grand manner as* PLATO *finishes speaking above.* PLATO *sees him.*)

PLATO (*aside to* EVERYMAN): Ah! Pythagoras—he's the head of a famous secret society, be sure you treat him with *great* respect.

(*To* PYTHAGORAS): Hail, master, here is one who would speak with you.

PYTHAGORAS: Let him approach.

EVERYMAN (*with great deference*): Pythagoras—master—Plato told me you could teach me many things.

PYTHAGORAS (*impressively*): I can. Number is the basis of *all* things. I have found number in sounds. I have heard the music of the spheres, I—but I impart my knowledge only to those who have the right to wear this sign.

(*Points to pentagram on his costume.*)

EVERYMAN: I see—but can you tell me who it was among your followers who found that there were numbers, incommensurables I think you call them, num-

bers that cannot be expressed exactly as whole numbers or fractions.

PYTHAGORAS (*startled*): How do *you* know that such numbers exist? That is one of our secret mysteries.

(EUCLID *enters from left.*)

PLATO: Euclid, here, wrote it in a book, along with all the other geometrical facts that had been discovered, and with some new facts he discovered himself.

(PYTHAGORAS *eagerly takes scroll from* EUCLID *and reads it attentively until curtain.* PLATO *continues, addressing* EVERYMAN.)

You'll study Euclid's book in its modern form some day. It's very slightly changed.

EVERYMAN: What good will studying your book do me, Euclid?

EUCLID: Once, long ago, a student of mine asked me that same question. I told the slave to give him a sixpence since he must have some thing from what he learned. It is true that only a few people will actually use what they study in my book, but everyone who studies it properly will gain some insight into the methods of rigorous logical thinking. In my opinion that is reason enough for studying it.

PLATO: Reason enough. And Euclid, Everyman, arranged the particular kind of geometry that he studied so well that few changes have been made in it.

Curtain

APPLICATIONS OF MATHEMATICS IN ANCIENT TIMES

Episode II

(PLATO *enters left with* EVERYMAN. ARCHIMEDES *is kneeling, tracing a figure on the floor and is obviously engaged in considering a problem.*)

PLATO: There is Archimedes, the greatest mind in science of the ancient world. He is working on a problem in geometry—he has drawn his figure in the sand. He is said to have been doing that when the Romans captured Syracuse in 212 B.C. One story has it that a Roman soldier came upon him while he was thus engaged and tried to take him to the Roman commander, Marcellus; but Archimedes refused to go until he had completed his solution of the problem. Thereupon, the soldier, not understanding, killed him; much to the regret of Marcellus, who greatly revered Archimedes' learning. Marcellus punished the soldier and showed great favor to the family of the scholar.

(ARCHIMEDES *sits back on his heels and contemplates his figure with an air of satisfaction.*)

He has finished his problem now, perhaps he will speak to us.

(*They go towards* ARCHIMEDES *who rises as they approach.*)

Greetings, Archimedes, will you tell my young friend, Everyman, something of your work?

ARCHIMEDES: With pleasure, though I can give him only a brief summary in the time I have—I did so many things. I am interested in many aspects of science connected with mathematics. I am a geometer and an analyst, a mathematician and an engineer. Most of my mathematical work is too difficult for those who are not especially trained, but you will be interested in hearing of some of my inventions, though they are not the most important part of my work. I wrote a treatise on levers and machines in general—you know the lever has tremendous possibilities. Give me a place where I can stand and I will move the world. You know, also, no doubt, the story of how I leaped from my bath crying "Eureka"—that means "I have found it" in your language-when I discovered a method for discovering the amount of gold in King Hiero's crown. He had information that the goldsmith who made the crown had kept some of the gold that was given him for the crown and put another metal in its place. I found that by measuring the amount of water displaced by an object I could tell what material the object was made of; since a given weight of any material would displace a definite amount of water. I invented machines which the Syracusans used in

fighting the Romans, though in truth, I did not invent them for that purpose, but merely as exercises in geometry. I worked with the screw and the spiral, and by means of a system of pulleys and cogwheels I was able to move a great ship loaded with freight and passengers over land. Everyone thought my work was wizardry of some sort, but it is very simple, I explained it all by mathematics.

(*During the recital* EVERYMAN *surveys* ARCHIMEDES *with wonder and awe. He is speechless.*)

PLATO: Truly a *very* brief summary, and not complete at that, Archimedes.

(ARCHIMEDES *smiles and shrugs, as he well knows he could never explain in a few minutes even a part of his work. He turns and kneels and begins working on a problem at the back.*)

Everyman, his work is staggering! There is only one other man who has ever done as much in mathematics, you will meet him later. Archimedes enriched even the highly developed Euclidean geometry, made important progress in algebra, laid the foundation of the scientific study of mechanics, and even anticipated the calculus. He reached a level which was not surpassed for two thousand years.

(VITRUVIUS *enters in a brisk business-like manner. He carries the posters under his arm.*)

PLATO: Hail, Vitruvius, where are you off to in such a hurry? Stop and show Everyman what you are carrying, won't you?

VITRUVIUS: I'm off to see to the work my men are doing on an aqueduct, Plato, but I'll be glad to stop and speak to Everyman for a minute.

(*To* EVERYMAN): I am Vitruvius, a Roman engineer and architect under the Emperor Augustus. You know we Romans don't care much about this abstract and theoretical work of the Greeks.

(VITRUVIUS *glances at* PLATO, *who shakes his head at such a deplorable statement.*)

We *did* things.

(PLATO *looks a bit disgusted, he believes in the ideal and not the material.* VITRUVIUS *is a trifle self-satisfied and scornful of the man of ideals.*)

These pictures show two of the architectural works of Egypt and Greece. Take the Pyramids for example:

(VITRUVIUS *holds up poster. He rolls out the figures in the following speech with evident enjoyment and great impressiveness.*)

Those Egyptians were technical experts, I tell you. Some of those stones weigh fifty-four tons. They cut them in the quarries on the other side of the Nile to exact size and shape, floated them down the river on barges, and then levered them into position by sheer man-power. The Great Pyramid is about 760 feet on a side (it would take about fifteen minutes to walk around it), it's about 484 feet high. The slope of the sides is 51 degrees and a bit more—and they did not have protractors! The measurements of all parts of the Pyramids show a scheme of careful proportional planning. There's much more I could tell you about the Pyramids, but I haven't time now. Here's the Greek temple—let Plato tell you about that, he used to live in Athens.

(*Holds other poster up—gives Egyptian one to* EVERYMAN *who rolls it up as he listens.*)

PLATO: The Parthenon! I'll not go into as much detail as Vitruvius did about the Pyramids. There is one thing, though, that I want you to see. The Greeks were great believers in harmony and all of their public buildings are in perfect proportion. Notice the sturdiness and the calm and repose of the building—the beauty of it! There is not a straight line in the temple. The engineers and architects did that in order to overcome certain optical illusions. You see a long straight line appears to sag, so they made a slightly curved line here (*Points to steps.*) to overcome that. But in addition to this, the Greeks realized that a curved line was beautiful of itself. Great engineering skill and great mathematical skill were needed to plan and build such temples with such perfect proportions.

(VITRUVIUS *takes poster from* EVERYMAN.

VITRUVIUS: I must go now—the aqueduct, you know. Good-bye, Plato—Everyman—I'm off.

(*He walks quickly off.*)

EVERYMAN: But wait, Vitruvius, you haven't told me anything about Roman engineering. Oh, Plato—

(*He turns disappointedly to* PLATO.)

PLATO: I can tell you a bit about their work. The Romans were not interested in science and not successful in truly scientific work. They were practical people. Rome did not produce a single creative man of science, not a single mathematician of eminence. They were highly efficient surveyors, map-makers, architects, builders, and engineers. Roman roads, bridges, aqueducts, and buildings have never been surpassed.

Curtain

ARITHMETIC

Episode III

(AHMES, PYTHAGORAS, A ROMAN, FIBONACCI, STEVIN, *are seated in a semicircle talking.* EVERYMAN *enters and comes up to them. They stop talking as he approaches.*)

EVERYMAN: I hope I'm not interrupting anything important. I am Everyman and I've just met some of the great geometers —oh, Pythagoras, I'm glad to see you.

PYTHAGORAS: Greetings, Everyman. These are a few of the men who worked with numbers—arithmetic, you would call it, but it is not like the arithmetic you are taught. You remember that I believe that numbers are the basis of all things. I investigated the theory of numbers and discovered many marvellous things. Here is Ahmes, an Egyptian scribe, who wrote the oldest treatise on mathematics that the world possesses.

(AHMES *hands* EVERYMAN *a scroll, he glances at it as* AHMES *speaks.*)

AHMES: Here is my manuscript, Everyman. You will find in it many problems and their solutions. I have dealt with fractions in arithmetic, many problems in what you now call algebra, and practical geometry. My papyrus is prized not only because it is the oldest mathematical work in the world (I wrote it about 3500 years ago), but because it shows how the Egyptians wrote numbers.

PYTHAGORAS: Your method of writing numbers made it most difficult to do calculations. But then our Greek way was not much better. We used our alphabet and each letter represented a number.

(EVERYMAN *returns scroll to* AHMES *with a nod of thanks.*)

ROMAN: Yes, your system was most cumbersome, but I marvel at the things you discovered with it. Our Roman numbers were better, though I must admit that sometimes calculations were most difficult. We used to use an abacus to help us do our figuring.

(*Hands abacus to* EVERYMAN.)

FIBONACCI: Those Roman numbers of yours weren't so easy to handle.

ROMAN: How now, Leonardo Fibonacci of Pisa, do you know any better ones?

(ROMAN *looks doubtful.*)

FIBONACCI: Yes, I do.

(ROMAN *shows surprise.*)

We call them Arabic numbers, though I believe they got them from the Hindus. I learned about these numbers from the Moors when I was in Africa where my father had charge of the custom house. I first introduced them into Europe in a book I wrote in 1202 called the *Liber Abaci*. It was a long time though before everyone adopted them—a very long time. Simon Stevin, I have heard you discovered something that made calculation even easier.

STEVIN: In my work as inspector of dikes, quarter-master general of the army, and minister of finance for Holland I found that by the use of decimals all computations could be performed by integers alone, even when they involved fractions. I wrote a book about it, and many people began to use them. Later, other men perfected the symbolism and now I understand that decimals are in common use.

PYTHAGORAS: They tell me that even young children are now able to make calculations with ease that formerly even great mathematicians had difficulty in performing.

Curtain

ALGEBRA

Episode IV

(DIOPHANTUS, AN ARAB, CARDAN, VIETA, RECORDE, *seated talking.* EVERYMAN *enters.*)

DIOPHANTUS: You are Everyman—Archimedes told me of you and the purpose of your visit. I am Diophantus. My book was the first work devoted chiefly to algebra. It contains the solution of equations and uses abbreviations for some of the words and operations, thus giving a kind of symbolism.

AL KHOWARIZMI: My algebra was based on the work of the Hindus, Diophantus. They did more in algebra than you Greeks.

DIOPHANTUS: We were at our best in geometrical work, but our algebra was of some importance, Al Khowarizmi. Your book was well known in Europe, in fact, the name algebra was derived from the title of your book.

AL KHOWARIZMI: Yes, they called the science of solving equations after my "al-jebr w'al muqabalah." My rule was put into verse which may be crudely translated:

Cancel minus terms and then
Restore to make your algebra;
Combine your homogeneous terms
And this is called muqabalah.

VIETA: In general then, you would say that al-jebr has as the fundamental idea the transposition of a negative quantity, and muqabalah the transposition of a positive quantity and the simplification of each member?

AL KHOWARIZMI: Yes, Monsieur Vieta, just that. The early Latin translators used my title in various forms, and in the 16th century it was finally applied to the whole science and not only to the solution of equations.

(*To* CARDAN *who is eager to speak.*)

Yes, Master Cardan?

CARDAN: I called my algebra the *Ars Magna*, the great art. It was devoted primarily to the solution of several types of algebraic equations. I have been told that it was the first step toward modern algebra.

VIETA: I knew your book. The trouble with algebra when I began to work on it in France around 1580 was that there was no definite method of writing it. Many before me, you among them (*To* CARDAN, AL KHOWARIZMI, *and* DIOPHANTUS) discovered many important things. I tried to improve the symbolism.

RECORDE: So you did—and many of the mathematicians who followed you did so, too. (*To* EVERYMAN) I am Robert Recorde and I wrote two text books: *The Grounde of Artes*, an arithmetic; and *The Whetstone of Witte*, an algebra, which were used in the English schools. You owe the equal sign to me, I first used it in the *Whetstone of Witte*. I called them twin lines and used them to denote equality because nothing could be more equal than twin lines.

VIETA: Most of the great mathematicians after us contributed many things to algebra. Compared with some other branches of mathematics it is a young science, but I'm afraid you won't have time to learn all of that now.

Curtain

MATHEMATICS IN THE MIDDLE AGES

Interlude

(GROSSETESTE *paces up and down reading a letter.* EVERYMAN *enters and speaks to him.*)

Greetings, sir, I am Everyman. Which of the mathematicians are you?

GROSSETESTE: Greetings to you, Everyman. I am not a mathematician. I am the Bishop of Lincoln, Robert Grosseteste. I was the first chancellor at the University

of Oxford about 1215. I was interested in science and I did everything in my power to teach it. I had a most promising pupil, Roger Bacon, you have heard of him?

(EVERYMAN *nods.*)

I insisted that mathematics was most important and the only true preparation for studying science. I wrote a book on the Calendar which had great influence I am told in bringing about the Gregorian reform.

EVERYMAN: You lived in the Dark Ages, didn't you?

GROSSETESTE: No age is ever dark—men have discovered that the more they study the so-called Dark Ages the more alive and bright they seem to be. It was the scholars of the Middle Ages who got manuscripts from Constantinople and from the Arabs and who translated them into Latin; thus preserving the learning of the Greeks. It is quite true that little original work was done in science—most people thought that whatever Aristotle said was final. I, myself, however advocated experiment and so did my pupil, Roger Bacon.

Curtain

THE FOUNDATIONS OF MODERN MATHEMATICS

Episode V

(DESCARTES *seated in chair.* EVERYMAN *enters.*)

EVERYMAN: Monsieur Descartes, I knew you from your picture. This mathematics is getting so involved and difficult that I can scarcely keep up with it.

DESCARTES: I understand. Mathematics in my time and in yours has gone far beyond the comprehension of the layman. Still, I can help you to some extent. You will meet three other mathematicians presently whose work laid the foundations of modern mathematics and modern science.

(NAPIER *enters.* DESCARTES *rises to greet him. Aside to* EVERYMAN.)

Here is John Napier, Laird of Merchistone, a Scot.

(*To* NAPIER.)

This, my lord, is Everyman, who desires to know something of your mathematical work.

NAPIER: (*acknowledging* EVERYMAN'S *bow.*) I devised a method of calculation that simplified computation and made it possible for astronomers and engineers to perform long and tedious operations with great ease. I called this short method the use of logarithms. Henry Briggs of England helped calculate the tables and made some improvements on my work. Has Descartes told you about his work?

DESCARTES: No, you came in just as I was about to.

(NAPIER *seats himself in chair.*)

I wrote a book on Analytical Geometry which combines algebra and geometry and makes it possible to translate facts about geometry into algebra and conversely. This makes the intricate and powerful machinery of algebra available for solving geometrical problems; and on the other hand, the geometrical illustration makes the algebra visible and concrete—But enough of that. Newton and Leibniz will be here shortly, you should know something about them. They had a tremendous quarrel once, but they have made peace now. Sir Isaac Newton is one of the greatest geniuses the world has ever known. His work is properly understood only by well-trained mathematicians and scientists, for it is the basis on which much of modern science rests, but it will be well for those not so well versed in mathematics to know something of Newton's work. Any one of his discoveries would have entitled him to undying fame—but *all* he did—he is superhuman.

(NEWTON *enters.* NAPIER *rises to greet him.* DESCARTES *shows him to a chair.*)

Tell Everyman of your work, Sir Isaac, if you please.

NEWTON: I won't do much more than give you the names—it would take far too much time to explain it all. I studied optics—invented the reflecting telescope, studied the spectrum, and discovered

many things about light. I investigated the question of gravitation by mathematics and was able to formulate the laws by which it works. I invented the fluxional calculus—a branch of mathematics which I needed in some of my investigations. I did some work in chemistry and made a table of the boiling points of many metals.

DESCARTES: Marvellous—truly marvellous!

NEWTON: If I have seen farther than you, it is by standing on the shoulders of giants. To myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

DESCARTES: Not all undiscovered, Sir Isaac. You discovered so much that you made it easier for those who followed you to push back the boundaries of the unknown.

(LEIBNIZ *enters. He greets all.*)

This is Everyman, Freiherr von Leibniz, he wants to know something of your work.

LEIBNIZ: I worked on calculus, too, and since my symbols were easier to handle, those are the ones that are used today. I made some improvements in the notation of algebra; then, too, I invented the first calculating machine.

EVERYMAN (*aside to* DESCARTES—NEWTON *and* LEIBNIZ *talk quietly to each other.*) Who invented the calculus?

DESCARTES: Let's not discuss that—that is what Newton and Leibniz quarrelled about. Probably they both discovered it independently of each other.

LEIBNIZ: I heard your question—we won't start that dissension again, but I'll say to you what I said to the Queen of Prussia once when she asked my opinion of Newton. When I consulted leading mathematicians on the Continent on certain very difficult points, I got no sort of help from them; but as soon as I wrote to Sir Isaac I received complete answers the next day. Sir Isaac was responsible for much the better half of all the mathematics that had been done from the beginning of the world down to his time.

Curtain

EPILOGUE

(*Spoken by* EVERYMAN *before the curtain.*)

I have come back to you at this point, not because there were no other great mathematicians, but because there were too many and because their work is so involved and complicated that it does not lend itself easily to discussion. Mathematics is the backbone of our present civilization, economic and scientific. With all these applications pure mathematics does not lag behind the practical; for there are mathematical associations and mathematical journals in most countries. Perhaps the best way to express my purpose in seeing these great men will be to quote what Professor Turnbull, the Regius Professor of Mathematics in St. Andrews (the oldest university in Scotland) said at the end of his book. You can substitute the word "play" for "book." Here are his words:

"And if this little book perhaps, may bring to some whose acquaintance with mathematics is full of toil and drudgery, a knowledge of those great spirits who have found in it an inspiration and delight, then the story has not been told in vain. There is a largeness about mathematics that transcends race and time: mathematics may humbly help in the market place, but it also reaches to the stars. To one, mathematics is a game (but what a game!) and to another it is the handmaiden of theology. The greatest mathematics has the simplicity and inevitableness of supreme poetry and music, standing on the borderland of all that is wonderful in Science, and all that is beautiful in Art. Mathematics transfigures the fortuitous concourse of the atoms into the tracery of the finger of God."

End

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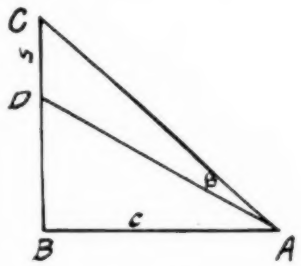
Calculus Versus Geometry

By CLAIRE FISHER ADLER
New York University
New York City

SOMETIME ago this problem was given on an examination for teaching positions in the high schools of New York City.



Given a right triangle ABC with CB a constant, CD a fixed length s, $\beta = \angle CAD$, $AB = c$. To show that when β is a maximum $s = 2c \tan \beta$.

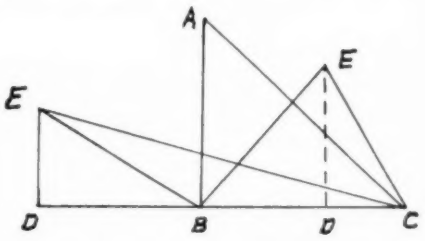
It seemed that the main difficulty in handling the problem was not in finding a solution but in finding the required geometric one for a problem involving the maximum value of a variable. Such treatments are usually inserted in an isolated section in textbooks on Plane Euclidean Geometry[1] and due to the limited time at the disposal of teachers of these courses this subject material is seldom touched; consequently, little is known to students in these courses about the treatment of such problems. Moreover in many cases these problems are handled somewhat laboriously and awkwardly by the geometric or purely analytic method whereas a calculus solution is extremely simple. For this reason it occurred to the writer that it would be an interesting novelty to compare the classic geometric solutions of some of these problems with the more elegant and powerful methods of the calculus. Two such solutions of each of the following four problems have been given, the first three of which are found treated geometrically in the texts before referred to[2] and the fourth is the problem mentioned at the beginning of this article.

PROBLEM 1

Of all triangles having two given sides, the area of that in which these sides include a right angle is the maximum.

Geometry

Let the triangles ABC and EBC have the sides AB and BC equal to EB and BC respectively; and let the angle ABC be a right angle.



To prove that $\triangle ABC > \triangle EBC$.

Proof: From E draw the altitude ED. The triangles ABC and EBC having the same base, $= BC$, are to each other as their altitudes, AB and ED.

Now $EB > ED$.

But $EB = AB$.

$\therefore AB > ED$.

$\therefore \triangle ABC > \triangle EBC$.

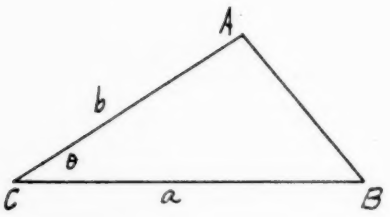
Calculus

Given triangle ABC with given sides $AC = b$ and $BC = a$ and these sides including $\angle ACB = \theta$.

Then area $S = \frac{1}{2}ab \sin \theta$.

[1] *Plane and Solid Geometry*, Wentworth, Revised Edition, Page 234.

[2] *Plane and Solid Geometry*, Ford and Ammerman (1919) Appendix, Page 206.



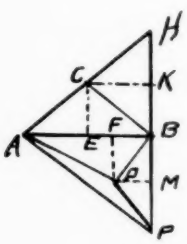
For a maximum $d_\theta S = \frac{1}{2}ab \cos\theta = 0$.

Therefore $\theta = 90°$.

PROBLEM 2

Geometry

Of all isoperimetric triangles having the same base the area of the isosceles triangle is the maximum.



Let the triangles ACB and ADB have equal perimeters and let AC and BC be equal and let AD and DB be unequal.

To prove that $\triangle ABC > \triangle ADB$.

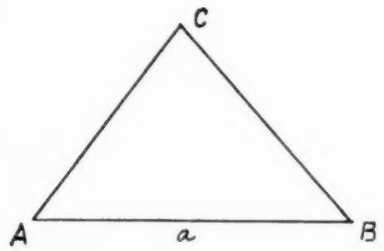
Proof Produce AC to H, making $CH = AC$; and draw HB. Produce HB, take DP equal to DB and draw AP. Draw CE and $DF \perp AB$, and CK and $DM \| AB$.
The angle ABH is a right $\angle$ for it may be inscribed in the semicircle whose centre is C and radius CA.
ADP is not a straight line, for then the $\measuredangle DBA$ and DAB would be equal, being complements of the equal $\measuredangle DBM$ and DPM respectively and DA and DB would be equal which is contrary to the hypothesis. Hence

$$\begin{aligned} AP &< AD + DP \\ &< AD + DB \\ &< AC + CB \\ &< AC + CH \\ &< AH. \end{aligned}$$

$\therefore BH > BP.$
$\therefore CE(=\frac{1}{2}BH) > DF(=\frac{1}{2}BP).$
$\therefore \triangle ACB > \triangle ADB.$

Calculus

Given ABC with $AB = a$ and, a given perimeter $= 2k$



Let $AC = x$.

Then $BC = 2k - (a + x)$.

Now area $S = \overline{K}\sqrt{(k-x)(a+x-k)}$

where $\overline{K} = \sqrt{k(k-a)}$

and

$$d_x S = \overline{K}\frac{\{(k-x)+(a+x-k)(-1)\}}{2S} = 0.$$

$\therefore \quad k - x = a + x - k.$

or

$$2k - (a + x) = x.$$

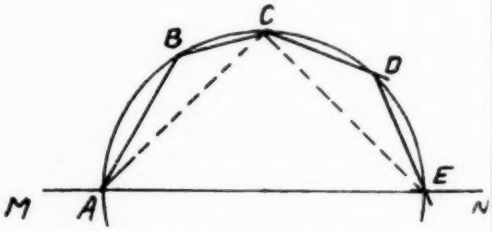
$\therefore \quad BC = AC.$

PROBLEM 3

Geometry

Of all polygons with sides all given but one, the one with a maximum area can be inscribed in a semicircle which has the undetermined side for its diameter.

This problem reduces to Problem 1 for a polygon with three sides. For four sides the area will be a function of more than one variable. The calculus discussion will be limited to this case.



Let $ABCDE$ be the maximum area of polygons with sides AB, BC, CD, DE and the extremities A and E on the straight line MN.

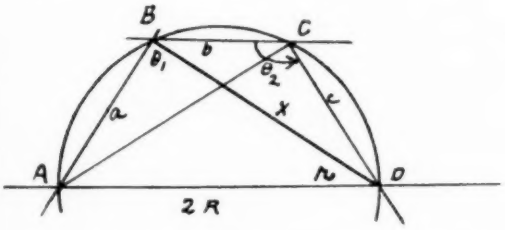
To prove that $ABCDE$ can be inscribed in a semicircle.

Proof From any vertex as C, draw CA and CE. The $\triangle ACE$ must be the maximum of all triangles having the sides CA and CE and the third side on MN: otherwise, by increasing or diminishing the $\angle ACE$, keeping the lengths of the sides CA and CE unchanged, but sliding the extremities A and E along the line MN, we could increase the $\triangle ACE$ while the rest of the polygon would remain unchanged; and therefore increase the polygon. But this is contrary to the hypothesis that the polygon is the maximum.
Hence, the $\triangle ACE$ is the maximum of $\triangle$ that have the sides CA and CE.
$\therefore ACE$ is a right angle.
C lies on the semicircumference; that is, the maximum polygon can be inscribed in a semicircle having the undermined side for a diameter.

Calculus

Let $ABCD$ be a polygon with sides $AB=a$, $BC=b$, $CD=c$ where a, b, c are fixed positive quantities.

To prove that $ABCD$ can be inscribed in a semicircle when $ABCD$ is the maximum polygon.



Proof Let $AD=2R$, a positive quantity; $\angle ABD=\theta_1$, $\angle BCD=\theta_2$, $\angle BDA=r$.

Then $S=\text{area}=\frac{1}{2}(ax\sin\theta_1+bc\sin\theta_2)$
where $x=\sqrt{b^2+c^2-2bc\cos\theta_2}$.
Hence S is a function of two variables. For a maximum,*

$$(1)\qquad \frac{\partial S}{\partial\theta_1}=0 \qquad \frac{\partial S}{\partial\theta_2}=0$$

also

$$(2)\qquad \left\{\frac{\partial^2 S}{\partial\theta_1^2}\,\frac{\partial^2 S}{\partial\theta_2^2}-\left(\frac{\partial^2 S}{\partial\theta_1\partial\theta_2}\right)^2\right\}>0$$

and

$$\frac{\partial^2 S}{\partial\theta_i^2}<0 \qquad i=1,2.$$

Now $\dfrac{\partial S}{\partial\theta_1}=ax\cos\theta_1=0 \qquad \therefore\ \theta_1=90°.$
$\qquad (x\neq 0)$

$$\therefore \tan\angle BAD=\frac{x}{a}.$$

$$\frac{\partial S}{\partial\theta_2}=\frac{bc}{2}\left[\frac{a\sin\theta_1\sin\theta_2}{x}+\cos\theta_2\right]=0.$$

But $\sin\theta_1=1 \qquad \therefore \dfrac{a\sin\theta_2}{x}+\cos\theta_2=0.$

$$\therefore \quad \tan\theta_2=-\frac{x}{a}.$$

Therefore θ_2 is the supplement of $\angle BAD=90-r$.
$\therefore \theta_2=90+r$.

But since ABD is a right triangle a circle may be drawn through ABD.

$\therefore$ arc $\overset{\frown}{BAD}=180°+2r$; hence arc $\overset{\frown}{BD}$ is the locus of vertices of all angles equal to $90+r$ having BD as a subtended chord. Hence C lies on this arc and the polygon $ABCD$ is inscribed in the circle having AD for a diameter.

Also

$$\left.\frac{\partial^2 S}{\partial\theta_1^2}\right]_{\substack{\theta_1=90°\\ \theta_2=\tan^{-1}-x/a}}=-ax$$

But since a and x are both positive $ax<0$.

$$\therefore \left.\frac{\partial^2 S}{\partial\theta^2}\right]_{\substack{\theta_1=90°\\ \theta_2=\tan^{-1}-x/a}}<0.$$

$$\left.\frac{\partial^2 S}{\partial\theta_2^2}\right]_{\substack{\theta_1=90°\\ \theta_2=\tan^{-1}-(x/a)}}=-\frac{bc}{2x}\left[\frac{\sqrt{a^2+x^2}+abc}{\sqrt{a^2+x^2}}\right]$$

$$=-\frac{bc}{2x}\left[\frac{2R+abc}{2R}\right]<0.$$

Also

$$\left[\frac{\partial^2 S}{\partial\theta_1}\,\frac{\partial^2 S}{\partial\theta_2}-\left(\frac{\partial^2 S}{\partial\theta_1\partial\theta_2}\right)^2\right]$$

$$=abc\left(\frac{2R+abc}{2R}\right)>0.$$

$\therefore ABCD$ is the maximum polygon.

PROBLEM 4

Geometry

Given a right triangle ABC with CB a constant, CD a fixed length s, $\beta=CAD$,

* *Granville, Smith and Longley.* Elements of Differential and Integral Calculus, p. 449.

$AB=c$. To show that when β is a maximum $s=2c \tan \beta$.

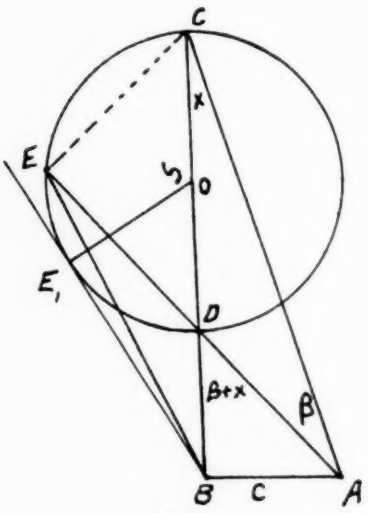


FIG. 1

Draw circle with center 0 and radius $s/2$ *Construction* Extend line AD until it cuts this circle at E. Connect B and E.

Let $\angle BCA = x$.

Then $\angle BDA = \beta + x$.

Now

$$\triangle EDC \sim \triangle BDA \qquad (3 \measuredangle =)$$

$$\therefore \quad \frac{ED}{DB} = \frac{DC}{DA}$$

also

$$\angle EDB = \angle CDA.$$

$\therefore \quad \triangle EDB \sim \triangle CDA$

$(\measuredangle =$ including sides $::)$

$\therefore \quad \angle BED = x \qquad$ and $\qquad \angle EBD = \beta.$

Now when E is at D $\angle \beta = 0$. As E moves from D to C $\angle \beta$ varies. When E is at C, β is again 0. Hence for some position of E on the semicircle β is a maximum. Consider the tangent line to the circle i.e. BE_1.

The $\angle E_1BC > \angle EBC$ since the secant line EB falls to the right of the tangent line E_1B.

Hence $\angle E_1BC$ is a maximum.

To prove When BE is tangent to the circle $S = 2c \tan \beta$.

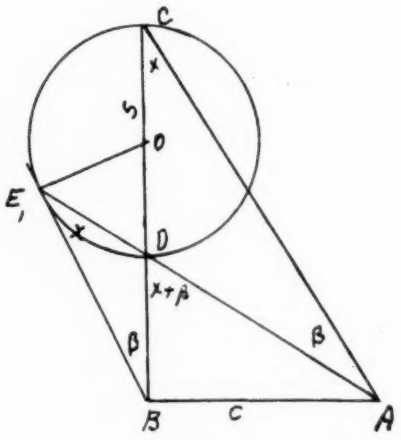


FIG. 2

$$\angle DE_1O = 90 - x = \angle E_1DO = x + \beta.$$

Hence

(1) $90 - x = x + \beta$; or $90 - \beta = 2x$ or $90 - (\beta + x) = x = \angle ADB = \angle DAB.$

$\therefore$

(2) $\triangle E_1BA$ is isosceles

$\therefore$

(3) $E_1B = BA = c.$

But

(4) $\tan \beta = \dfrac{E_1O}{E_1B} = \dfrac{s/2}{c}.$

$\therefore$

(5) $s = 2c \tan \beta.$

Calculus*

Using Fig. 2. as above and letting $DB = h$.

(1) $$\frac{s}{\sin \beta} = \frac{AD}{\sin x} = \frac{h}{\sin x \cos (x+\beta)}$$

(2) $\therefore \quad s \sin x \cos (\beta + x) - h \sin \beta = 0.$

Differentiating and solving for $d\beta/dx$

(3)

$$\frac{d\beta}{dx} = s\left[\frac{\cos (\beta+x)\cos x - \sin (\beta+x)\sin x}{s(\sin x)\sin (\beta+x) - k\cos \beta =}\right].$$

* This problem is the classic picture or tapestry problem found in many standard calculus text books. See Problem 17. P. 114 in reference given in Problem 3.

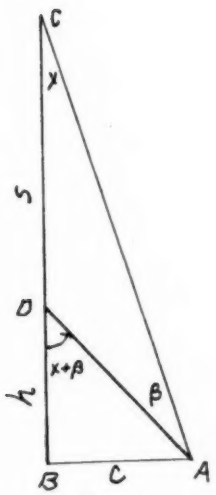


FIG. 3

Hence if

$$\frac{d\beta}{dx}=0$$

$$\cos(\beta+x)\cos x-\sin(\beta+x)\sin x=0$$

$$\therefore\quad \cos(\beta+2x)=0.$$

(4) $\therefore\quad \beta+2x=90^\circ \quad \therefore\quad \sin 2x=\cos\beta.$

But

(5) $$\frac{s}{\sin\beta}=\frac{c}{\sin x\sin(x+\beta)}=\frac{c}{\sin x\cos x}$$

$$=\frac{2c}{\sin 2x}=\frac{2c}{\cos\beta}.$$

(6) $\therefore\quad s=2c\tan\beta.$

A comparison of these four problems reveals the superiority of the methods of the calculus in the second and fourth problems only. In the first problem geometry was declared more satisfactory by students in a course in general mathematics where very elementary ideas of calculus had been introduced. The superiority of the geometric solution is demonstrated in problem three where even a comparatively simple application of the calculus leads to rather lengthy computations. However in the fourth problem the efficiency of the calculus is clearly demonstrated. It has led to a neat and clear solution, without the introduction of any special device, lengthy explanations of construction, discussion or detailed proof. It is surely a problem admirably adapted to delight the ardent adherents to the methods of the calculus!

THE ALGEBRA OF OMAR KHAYYAM

By DAOUD S. KASIR, PH.D.

THIS work presents for the first time in English a translation of the algebra of Omar Khayyam. In an introduction the author traces the influence of earlier Greek and Arab achievements in the field of mathematics upon the algebra of Omar Khayyam and in turn the influence of his work upon mathematics in Persia in the Middle Ages. The translation is divided into chapters, and each section is followed by bibliographical and explanatory notes.

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Significance, Meaning, Insight—These Three*

By B. R. Buckingham

Editorial Department
Ginn and Company, Boston, Massachusetts

Five or six people devoted to education and hence of more than average intelligence are having luncheon. Each has ordered a different meal; each is expecting to settle his own account. The waiter, however, totally disregarding the attempt which has been made to secure separate checks, has brought in but one bill and has placed it before the most prosperous-looking person in the group. You have doubtless participated in such a luncheon. If you have, you remember the comedy which ensues as these table companions try to break down the total into the proper amount for each person and assess the tip which each should pay. Meanwhile the waiter who caused all the trouble hovers about, bored or tolerant or supercilious, according to his nature. The comedy is rendered farcical by the fact that everybody talks, nobody thinks, and nobody has the right change.

If you play contract, you have, no doubt, noticed that certain people, generally not very good players, seem unable to manage the number combinations whose sums are thirteen. They may not be able even to count the tricks already taken unless they point to each one as they count. Moreover, they have to touch them; and then—caution being engendered of experience—they begin at the other end and count once more as a check.

Yesterday we read in the newspaper that Secretary Wallace had lately spent three hundred thousand dollars for a million pigs. The A. A. A., we were told, was buying up surpluses and distributing them to the unemployed. If I were to ask—perhaps not of you, but of a general audience—how much Secretary Wallace was paying per pig, there would be a good deal of fumbling about as to whether one should divide three hundred thousand by a million or vice versa. And even those who might see that money must be divided by pigs, (or shall we yield to the pedants and say the number of dollars divided by the number of pigs?), even those might yet bestow a roving commission upon the decimal point and accept results ranging from thirty dollars to thirty cents.

If these are reasonable statements of typical events,—and I think you will agree that they are not greatly overdrawn—then we are forced to conclude that the teaching and learning of arithmetic in our public schools have failed dismally to assure a widespread intelligence in the use of number.

In a certain school system of high standing in its state a survey of arithmetic is now taking place. The school people have found that after continuous trial of a certain type of procedure, the results are unsatisfactory. The system which has been in use is one which enthrones drill. According to this system, if your pupils are not up to standard in the third, the fifth, or the eighth grade, you did not drill enough. You should have begun earlier, devoted longer periods to drill, bought and consumed more drill pads, and continued to do this without intermission. The remedy for disappointing results is therefore to drill harder and faster and longer than you did before. Having found that your prescription is no good, you are bidden to give larger doses of the medicine which has produced the disorder.

In the survey of arithmetic to which I am referring, the teachers have formulated questions which they want answered. Some are old queries which teachers have

* An address delivered at the Eighteenth Annual Meeting of the National Council of Teachers of Mathematics at the Palmer House in Chicago, Ill., on February 20, 1937.

been asking for ages when they had a chance. For instance, you will recognize this: "Why is it that pupils who come up to us from the lower grades do not know their arithmetic better?"

Here is one, however, which offers more promise. It is from a junior-high-school teacher. "Many science teachers use methods that are not taught in arithmetic or use formulas which seem different to pupils. Is it not possible for mathematics teachers and science teachers to teach similar things by the same methods?" If you give that question thoughtful consideration, you will soon find yourself deeply involved in the problem of understanding, of something that stands behind the particular devices on which the mathematics teacher and the science teacher fail to agree. What do you think of the suggested solution, namely, that arithmetic teachers in the elementary school and science teachers in the secondary school do the same things in the same way?

Another question: "Is it not possible for teachers to consider some of the attitudes which pupils have toward mathematics?" There are some wrong attitudes which might be considered. One pupil says he never liked mathematics, never could do it. Another says his family were all poor in mathematics; so he cannot be expected to be a good student in that subject. Another says that if the answer is right, what difference does it make what method is used? Here, then, is a teacher who dares to raise the question, independently of accuracy and computation, whether certain attitudes might not be expected as the result of learning and teaching in the elementary grades.

What do you think of this one: "Mathematics depends upon abstract concepts. It is rich in symbolism. It has certain rituals and procedures. Do pupils feel these things?" This is a call to teach arithmetic as mathematics, to give it meaning as a branch of that science. This seems pretty far from a daily stint of mixed drill or speed practice in column addition.

Finally, again and again among the questions occurs a type for which these may stand: "Can we, by means of arithmetical problems, help children to gain power in reasoning?" "My children can do abstract work but they cannot solve problems; what can I do?" You have heard such questions before. Apparently computational skill, no matter how highly developed, is not enough. The pupil must know how and when to use number under varied social conditions.

These questions serve to show how in one excellent school community an early and long-continued drill technique has led to a desire for something which such a system cannot give—for an understanding, so that symbols and formulas that are only apparently different will not confuse; for a favorable attitude toward arithmetic; for a conception of arithmetic as a mathematical system; and for an application of arithmetic as a way of thinking. There is a demand—and I think it is greater today than ever before—for what I may call sense in mathematics and for a favorable emotional response to it, a willingness as well as an ability to look at things number-wise.

At our professional schools most of us were taught Thorndike's three laws of learning: readiness, exercise, and effect. The law of exercise was the law of repetition, the law under which drill found its place. Of late the validity of that law has been questioned, and by no one more than by Doctor Thorndike himself, who, as you probably know, experimented with sheer repetition, devoid of any concomitant, and found there was no learning at all. One type of experiment was to blindfold a person and tell him to draw a line four inches long and repeat again and again. The results were what you would expect. No improvement in drawing a four-inch line resulted from such repetition. The law of exercise came into being because something that usually accompanies repetition gave repetition its apparent value. The repetition itself, devoid of under-

standing, devoid of knowledge of progress, devoid of any critical sense, is of almost no use at all.

Along with the law of exercise we were, many of us, brought up on an atomistic psychology—a psychology in virtue of which we divided and subdivided arithmetical processes into types and unit skills and undertook to teach each minute part in the conscientious belief that only so could the process as a whole be mastered. This atomistic psychology is giving way to one which emphasizes not the parts but the whole—in short, to an organismic psychology. Singularly enough, we have had in our hands for over forty years an organismic psychology of arithmetic. McLellan and Dewey (John Dewey was then a young man) issued *The Psychology of Number* in 1895. It was an important book in its day, and it is quite as important now. It has never been superseded. It taught us, though we were slow to learn, the importance of the whole. The authors said, in substance, Let us begin with wholes, because they give significance to parts. Let us not believe that we should begin with parts and that the pupil can in some way put them together to make meaningful wholes.

The whole is not, in organismic psychology, merely the sum of its parts. However true that may be in mathematics, it is not true in psychology of the organismic type, and it is not true even in life. At some critical point the summation of parts becomes something besides the sum. We are thinking a great deal these days about the Supreme Court. If we have five judges on one side, we have something more than the sum of the parts of five, something more than five ones, or three and two, or four and one, something more than merely five as a summation. We have a *decision*, something that changes utterly the significance to the people of the vote of the judges.

Before going farther, I want to come to an understanding with you with reference to terminology. I have one or two suggestions to make in that respect. I have used the terms *significance*, *meaning*, and *insight* in the title of this paper. By the *significance* of number I mean its value, its importance, its necessity in the modern social order. I mean the role it has played in science, the instrument it has proved to be in ordering the life and environment of man. The idea of significance is therefore functional.

On the other hand, the *meaning* of number, as I understand it, is mathematical. In pursuit of it we conceive of a closely knit, quantitative system. It has broader aspects than those which we teach to children. The domain of number transcends the field of arithmetic and has become so diverse that mathematicians no longer give us a definition of number. Its domain lies beyond whole numbers and fractions, positives and negatives, irrationals and imaginaries, etc., each classification being infinite in its members. Even in the realm of so-called natural numbers the series is infinite. "There is no last number."

Under the heading of meaning I include, of course, the rationale of our number system. The teacher who emphasizes the social aspects of arithmetic may say that she is giving meaning to numbers. I prefer to say that she is giving them significance. In my view the only way to give numbers meaning is to treat them mathematically. I am suggesting, therefore, that we distinguish these two terms by allowing, broadly speaking, significance to be social and meaning to be mathematical. I hasten to say, however, that each idea supports the other.

By making this distinction between significance and meaning I think we shall gain greatly. We shall perhaps begin to do two things, namely, to teach arithmetic as a social study and to teach it as mathematics. The one emphasis will exalt arithmetic as a great and beneficent human institution, the supporter of a fine humanistic tradition. The other emphasis, the mathematical one, will lift arithmetic, even in the primary grades, from a

formalized symbolism to the dignity of a quantitative system. In short, when arithmetic is taught with meaning it ceases to be a bag of tricks and becomes, as it should be, a recognized branch of mathematics.

It seems to me entirely possible to offer real arithmetic successfully in the primary grades, provided it is concrete. This arithmetic can meet the most exacting mathematical demands. It can be a type of arithmetic which does not have to be done over, which necessitates no new moves designed to correct errors or misconceptions. Concrete arithmetic is, or can be, true arithmetic. It is the true and proper support of abstract and generalized arithmetic. The trouble is that in the primary grades we do not give enough concrete arithmetic. We begin with counting objects. This is admirable so far as concreteness is concerned, though it is generally unnecessary, because most children can count objects before they receive instruction in school. Usually the child is then expected to take up the number facts as abstractions. This he is not ready to do. We show too great haste in arriving at a goal which we see clearly, but to which the child has not been led by steps that conform to the way he grows.

When I speak of *insight* I am talking no longer about arithmetic, no longer about the curriculum, but about the learner himself. Insight implies a person. When significance and meaning are given to arithmetic in the presence of a class, the pupils gain insight. They do so simply because that is the way man, the human animal, is made. He is essentially an experiencing organism. Bagehot in one of his literary essays explains Shakespeare as the product of a first-rate experience operating on a first-rate experiencing nature. This is the distinction I want to make. The school should offer a first-rate experience. The child brings to it, by the very nature of his organism, an experiencing nature.

When we confront children with a significant and meaningful experience, and when they make this experience theirs, they acquire insight, each to the degree that he is able. More specifically, and in particular relation to number, they gain in two ways: they understand number as such, and they also understand when and how to use it to serve their purposes. It is obvious that significance and meaning are nothing without an experiencing nature.

The more I think about arithmetic, the more firmly I become convinced that in order to carry out the high purpose of the subject we must consider the value of the stretched-out curriculum, that is, of the curriculum which in many respects begins earlier and lasts longer. I would begin arithmetic in Grade I. The idea of postponing all arithmetic until Grade III or even later flies directly in the face of the doctrine to which I am referring. Children come to school with a far greater experience with number than has until recently been supposed to be the case. When I first went into this matter, no one was sure that the child upon entering school knew anything about numbers greater than 4. In fact, G. Stanley Hall, summing up the knowledge on this question that existed about 1910, stated that the six-year-old child "knows the numbers up to 4." This is an extraordinarily incomplete statement.

The child of six has already used a great deal of concrete arithmetic. He can count; he can make up a group of six or eight objects; he can identify a given group as consisting of six or eight objects; he can perform additions of one-place numbers if the data are presented concretely or in problem form. He will be pretty likely to give you the correct answer to such a problem as "If you had 5 cents and your father gave you 3 cents more, how many cents should you have then?" He is "ready" for instruction.

This is not the place to give the interesting statistics as to the ways in which children exhibit different types of number abilities before instruction at school. No

doubt many of you are already familiar with the evidence to the effect that upon entering school children are already using number for their own purposes. I know of no further necessary criterion as to the grade-placement of topics in arithmetic than these criteria of readiness and use.

A word more about readiness. As I have indicated, the attack on the law of exercise has been vigorous and successful. No one, however, has ever attacked the law of readiness. The whole investigation of the Committee of Seven has been carried out on the assumption that this law is valid. As I understand the law in this connection, it leads me to believe that you must begin arithmetic when the child enters school for the simple reason that he is then ready for it.

Furthermore, we should definitely abandon throughout the elementary school the high degree of compartmentalization to which we have hitherto been committed. This compartmentalization has led us to identify each grade with a particular topic: addition and subtraction with Grades I and II; multiplication and division with Grades III and IV; fractions with Grade V; decimals with Grade VI. The organismic idea in psychology would suggest that, for example, instead of devoting Grade I to certain addition and subtraction facts, we offer a more varied program. Instead of postponing fractions to Grade V we may properly begin them in Grade I. Miss Polkinghorne gives good evidence that children are already using fractions when they enter school. Denominate numbers may properly have a place in every grade in the elementary school. Decimals may come earlier in the course than some aspects of common fractions. And number concepts should be developed early and employed throughout the course.

Broadly speaking, there are only two kinds of functional concepts of numbers: one is reproduction, and the other is identification. When the child acts under the guidance of number, (and the same may be said of the adult,) either he is acting in response to a number idea, or else, being confronted by a situation, he identifies it as to its number aspect.

At my hotel this morning I asked the clerk at the information desk, "How do I get to the Palmer House?" She was too experienced to say, as some might have: "Well, now, you go out the main entrance and turn left. The first street you come to is Congress Street, and the next is Jackson. You do not turn on Jackson, but go on to Van Buren. And you don't turn there either; but when you come to Monroe, you turn left." On the contrary, the clerk said, "Six blocks north, and one west." Here were number ideas; and acting under their guidance I duly arrived at the Palmer House. In my actions I was reproducing these numbers. The child does the same thing in the various grades of the elementary school. He is called upon to act under the guidance, first, of small whole numbers, then of larger whole numbers and fractions, then of percentages, and so on. At about the same time he is expected to recognize the number elements in a situation and to identify these elements. When so identified, the ideas thus produced may themselves constitute the basis of a new reproduction. Thus a course of action may consist of a series of identifications and reproductions.

Number concepts in the early grades, if they are to be rich and vigorous, must be handled with meaning. That, as I have said, is the mathematics of the question. For example, how shall the child know with meaning the number 6? Obviously he may know it from its position in the number series; but in order that the knowledge may be meaningful, we must be careful that this position is apprehended as a quantitative one. Mere rote counting has no quantitative meaning. A child should, of course, understand that 6 is one more than 5 and one less than 7; but if his rote counting has no more quantitative aspects than his saying of the alphabet, there is no more reason to ex-

pect him to give numerical meaning to 6 than there is to expect him to give numerical meaning to F or K or P.

Six must not only be learned as having a position in a quantitative series; it must also be learned in its relation to other numbers than those adjacent to it in the counting series. It is 3 and 3, 4 and 2, two 3's; it is what you get when you subtract 4 from 10; and it is many other things.

Another, and still better, way of knowing a number is to know it as a ratio. But I am sure I do not need to pursue these rather obvious matters farther. The richness of meaning that the ratio idea permits needs no further comment on my part.

Number concepts are not, of course, to be restricted to whole numbers. Fractions must also be taught with meaning. Children—and, for the most part, teachers too—do not have a lively sense of the meaning of fractions. I well remember the astonishment of a class of teachers when I told them that 2/3 was not merely 2 of the 3 equal parts into which a unit has been divided but also 1 of the 3 equal parts into which 2 has been divided. For certain purposes it is especially desirable that the numerator of a fraction should be taken as indicating the size of the quantity which has been divided. Again, the meaning of a fraction as a ratio is feebly represented in our teaching.

One of the troubles in all this is that having found certain things in arithmetic to be difficult, we have taken them out of the course. I submit that there is another alternative, namely, to teach them with meaning and significance.

For example, one of the things which we have pretty generally taken out of the arithmetic course is proportion; yet proportion is profoundly useful as a form of thought. It is one of the great ideas of arithmetic. Indeed, it is almost the only idea which permits one to project one's past experience into the future and make a reasonable estimate of what is to be expected. We took proportion out of the course because it was too hard; and it was too hard because we failed to lead our pupils into its meaning. To do this, something more is required than teaching the meaning of proportion at the time it is first brought up. Meaning cannot be taught merely *ad hoc*. The sources of it lie farther back.

When a teacher asks how she can rationalize the meaning of the fraction in the fifth grade, or how she can teach her pupils to solve problems in the sixth grade, we must tell her that the complete accomplishment of her purpose can only be secured when the first simple rudimentary whole ideas about these things are introduced in the first and second grades. It is from this point of view—to say nothing of others—that the teaching of number in Grades I and II is so important. That is where meaning and significance first give rise to insight. That is where, on a concrete basis, we lay our foundations for fractions and whole numbers, for counting processes and measurements. That is where we begin the steps by which we may succeed later in explaining long division, multiplication by a fraction, and the placing of the decimal point in division.

I ought not to take more of your time. In general, I regard as a suitable course in arithmetic one which begins in Grade I and teaches all those things for which the child is then ready. It will begin every topic in arithmetic in a rudimentary form sooner than it does now and will not bunch each topic in a given grade or semester. It will postpone some topics which now occur too early—long division, for example. It will meet the child where he is with concrete material, and will generalize only as he matures.

It will likewise avoid confusion by having one standard method of performing each process. If you have read Ballard's book, you will remember his doctrine of the King's Highway. It is a broad, easily traveled road. From one point to another it may not always be the shortest way.

There are byways that may not take so long to travel if you know how to do it; but these byways are treacherous, they abound with difficulties, and only those who know the country well dare to travel them. Ballard counsels us to teach division by one rule rather than two, and what he means is that all division be written out in the long form. He would teach interest by one method and not by three or four. He would teach the solution of a quadratic equation by one method. The King's Highway, then, turns out to be, in less poetic form, the one general method which will always work. Afterward we may explore the country and investigate the advantages of its byways and short cuts. We shall teach short division, but we may not do so until the sixth grade. We may also teach alternative methods of computing interest after the one general method which always gets there has been learned.

When a teacher asks, "How can I make children understand the division of fractions?" or "How can I teach problem-solving?" her questions require answers that must be sought in distant places. They are seldom found in methods to be used at the moment. They rest upon a slow development of significance and meaning, perhaps over a period of several years. If pupils have not understood what they have been doing from the first with easy material, if no insight has aided them in learning, they cannot understand by any act of yours or theirs applied later to more difficult material. They must go back and build up the meaning and significance of the numbers, whole and fractional, which they employ. Only then will they be likely to understand division of fractions. Only then, with a varied experience in the use of numbers and a sense of their inner meaning, can we expect the insight which will ensure ability to solve problems.

So the words I give you this morning are significance, meaning, insight, these three; but the greatest of these is insight.

Notice to Subscribers!

The Mathematics Teacher wishes again to ask subscribers to bear with us until our mailing list is all straightened out. The confusion caused by the death of Miss Winspear and the large volume of new subscriptions that have been pouring in this fall has almost swamped our office but we now feel that we are almost "out of the woods." We were compelled in the beginning to get help that was not familiar with our methods and this has caused some trouble. However, you subscribers can help us greatly if when you send in $2.00 for a subscription you will indicate whether you are a new subscriber or whether you are renewing your subscription. You can also help us if, when you change your address or (in the case of the women get married), you will send us your name as it formerly appeared on our mailing list, your correct old and new addresses, and also your new name where it has been changed. We have been caused no end of trouble by those who have failed to do these things and also by those who do not pay their subscriptions or else ask us to drop them from our list when we send them expiration notices.

Random Notes on Geometry Teaching

By HARRY C. BARBER

Note 4
Superposition

THERE has been much discussion of what should be done with the superposition proofs. The plan often advocated is to postulate all the theorems usually so proved. In England the reason given is that when we move a figure we cannot be certain that it does not change in size and shape. In the United States the reasons given are that superposition is not a suitable method at the outset; it is not readily understood by the beginner, and it interferes with the later use of other methods of proof.

The fundamental fact that ought to help us to decide what to do about the superposition proofs is this: *all direct measurement is a process of superposition.* In direct measurement we superpose a unit of measurement upon the thing to be measured.

This at once disposes of what is referred to above as England's argument against superposition. Of course we can take an agnostic position in any field of thought, but if we cannot know congruence by superposition then we cannot measure anything and there is not enough of elementary geometry left to bother about. But if we are going to talk about measurement at all then what we need to do is to define congruence by superposibility—make superposition the test of congruence—and go ahead. Our logic is then impeccable.

Now for the "United States arguments": It is clear that the idea of superposition is familiar to the pupil when he first comes to the geometry class. Ask him to measure some distance or to test the equality (congruence) of two blocks, and he will immediately superpose. While he may not be articulate about it, he knows that in real life superposition is the basis of all measurement. In geometry he is going to learn something about indirect measurements. These are based on direct measurements but they need theorems to justify them or to tell us how to go about them.

If, then, measurement starts necessarily with superposition and if superposition is familiar to all beginners in demonstrative geometry, then it is a sound method with which to begin geometry's study of measurement.

If superposition is approached in this spirit it can easily be understood by beginners in geometry. First there should be a little *actual* superposition, then there should be informal superposition of ideal figures some of which *do not fit.* Then there should be mentioned and illustrated the three kinds of motion suggested by the words slide, rotate, and overturn. When this has been done, the pupil is ready for the formal proofs.

To omit the superposition proofs at the beginning of plane geometry is to miss the strongest link we have between everyday experience and the argument of geometry. Here is a place where teachers need to do battle against a current unfortunate trend.

As to the pupil's disinclination to change from superposition to other methods of proof: Of course when one knows only one method of proof one is inclined to use it, but experience shows that the three words Prove Without Superposition, and one model proof, placed at the head of the set of exercises next following the superposition proofs, immediately obviates any difficulty of this kind.

And finally, the carpenter superposes, the dressmaker superposes, the civil engineer with his transit and tape superposes, all direct measurement proceeds by superposition. Where else then should the argument about geometry's measurement begin if not with superposition?

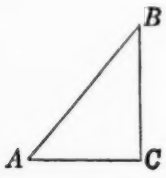
◆ THE ART OF TEACHING ◆

Imagery in Numerical Trigonometry

By ELLIOTT H. KAUFMAN,
Jackson Intermediate School, Detroit, Michigan

STRANGE as it may seem, one of the most difficult concepts for the pupil at the junior high school level to grasp in the unit on numerical trigonometry is the location of the three sides of a right triangle, the opposite, the adjacent and the hypotenuse.

The author has found it helpful in his classes to substitute for mere words, word-pictures when locating these sides. This may be accomplished by taking the right angle ABC: (demonstrate)



Imagine standing enclosed in a triangular corral with a bow and arrow. You are standing on point A (Angle A). You are facing what ever is in front of you—in this case, BC. You shoot your arrow directly forward. Where does it strike? Naturally it hits the side opposite you, BC. Now, take your position on point B (Angle B). Face forward. Shoot as you stand. Where does it strike? In this case, it hits the opposite side to Angle B—side AC.

So that, you see when Angle A is concerned, BC is the opposite side; when Angle B is concerned, AC is the opposite side. Now you have left two other sides. One side is AB which you will notice is opposite the right angle C. This side is called the hypotenuse. No matter how it is labelled, this side is *always* the hypotenuse. It doesn't matter whether your problem concerns side BC or side AC, side AB, or whatever it may be called is always the hypotenuse.

You have learned now that if you are talking about Angle A, side BC is concerned, that is, the *opposite* side; if it is angle B, then AC is opposite and that whether it is Angle A or Angle B, side AB has remained the same—the hypotenuse. There is only one side left, AC, with Angle A and BC with Angle B. This is called the adjacent side. We now have our three sides. Angle A is called for in our problem. Then BC is opposite, AB is the hypotenuse, and the only side left, AC is adjacent. Angle B is called for in our problem. Then AC is opposite, AB is still the hypotenuse, and the only side left, BC is adjacent.

When recognition of the sides is accomplished, it becomes much less of a task to teach which ratio to use in solving sine, tangent and cosine problems.

Another everpresent difficulty which faces the junior high school pupil is the remembering of the ratios themselves. All of us remember our own college days and the devices, silly in themselves, which we used to memorize various lists in Latin or History or English or what-not. Why not, then, use a similar device to teach the ratios? We have found that the following one is helpful:

My father smoked cigars. The name of these cigars were SINCOS. They were tan in color. He liked to smoke them so well that with each puff of smoke which he exhaled, he exclaimed,

"OOA AHH! OOA AHH!"

Now, if we put down what he said like this: (demonstrate at board)

$$\begin{array}{ccc} O & O & A \\ A & H & H \end{array}$$

and draw lines across, and up and down like this:

$$\begin{array}{c|c|c} O & O & A \\ \hline A & H & H \end{array}$$

we have the three ratios: for the tangent, opposite over adjacent; for the sine, opposite over the hypotenuse; for the cosine, adjacent over the hypotenuse.

Does this sort of teaching in a staid subject like numerical trigonometry shock my academic-minded fellow teachers? Hughes Mearns tells how he teaches in his classes the rules for the multiplication of signed numbers:

"Write down on the palm of your *LEFT* hand in ink this rule:

$$\begin{array}{l} + \cdot + = + \\ - \cdot - = + \\ + \cdot - = - \\ - \cdot + = - \end{array}$$

Go home with this on your hand. Do not wash your hand. Tell your mother that you can't wash your left hand and tell her why. When you have learned the rule, then wash your hand."

The Tenth Yearbook

of

The National Council of Teachers of Mathematics

on

The Teaching of Arithmetic

was recently voted one of the sixty most outstanding contributions of 1935. It discusses the most important issues with reference to the teaching of arithmetic at the present time, such as the problem of transfer, the mechanistic approach versus the understanding approach; the place of the activity program, informal versus computational arithmetic, current classroom practice, etc. The writers are all outstanding authorities in the field.

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IN OTHER PERIODICALS

By NATHAN LAZAR

Alexander Hamilton High School, Brooklyn, New York

1. Bartlett, M. S. " 'Natural' Mathematics." *The Mathematical Gazette.* 21: 44–45, February, 1937.

The earlier introduction of a course in statistics is urged since it is almost a necessity for workers in any of the observational sciences. Moreover, the students of those sciences might be helped if they were presented with a more pragmatic view of the mathematics they require.

2. Bedell, Ralph C. "A Methodology of Guidance for Teachers of Science and Mathematics." *School Science and Mathematics.* 37: 968–976. November, 1937.

Teachers of mathematics and of the sciences have not done much in adjusting the curricula in their subjects to the individual interests, abilities, aptitudes and needs of their students. The essential points of a methodology of guidance are as follows:

(a) "Measurement of the factors that influence success in science and mathematics.
(b) "Diagnosing each student in terms of these factors and his needs.
(c) "Prescribing an education in terms of the diagnosis.
(d) "Continued counselling to check the effectiveness of the prescription, the accuracy of the diagnosis and to make such modifications as seem necessary."

The specific application of each of the above steps is given to the problems of the teachers of mathematics and of the sciences.

3. "The Board of Education Summer School for Teachers of Mathematics in Technical Colleges, July, 1936." *The Mathematical Gazette.* 21: 40–43. February, 1937.

A report by a "Member of the Course" of the various lectures that were held for the first time at Oxford for those who teach mathematics in technical colleges. It is extremely gratifying to find that our British brethren, too, are discovering that even in the colleges it is as important to know *how to teach* as *what to teach.*

4. Braverman, Benjamin. "The Indirect Proof in a Life Situation." *High Points,* Vol. 19, No. 6. June 1937, pp. 77–78.

"One of the encouraging tendencies in present day teaching of demonstrative geometry as compared with that of a generation ago is the greater emphasis placed on the indirect method of proof. Teachers today realize that, primarily, demonstrative geometry is taught for the purpose of giving pupils an introduction to logical reasoning, and that the indirect proof, because it is used so often in life situations, offers the progressive teacher a rich opportunity to connect geometry with life."

After outlining the essence of the indirect proof the author gives two illustrations. The first is an extract from the story "The Sign of Four" from the *Complete Sherlock Holmes.* The second is the substance of an argument presented by an attorney to prove that his client is innocent of the crime of which he is accused.

5. Butterweck, Jos. S., "How Much Progress in Secondary School Geometry?" *School Science and Mathematics.* 37: 911–919. November, 1937.

In order to find an answer to the above question, the author examined thirty-three geometry textbooks, selected at random from a textbook library, ranging from the year 1826 to the year 1932. He analyzed them from the following points of view:

(a) method of introduction
(b) organization
(c) exercises used
(d) historical references
(e) methods of proof
(f) authorship

He arrived at the following pessimistic conclusion: ". . . Unless drastic changes can and will be made in the textbook organization to bring the subject in line with modern principles of psychology and education, a functional knowledge of geometric principles will not result from the study of geometry. . . . If geometry cannot be made a creative adventure for youth, the secondary school had better send it the way taken by Greek during the past generation."

6. Edington, Will E. "Vitalizing Mathematics." *National Mathematics Magazine.* 12: 27–28. October, 1937.

The following five suggestions are made for the vitalizing of mathematics:

(a) "Conscious, continual and consistent reference to the analogies between the mathematical processes and the ordinary processes met with in the work-a-day world.

(b) "Dissemination of information on the application of mathematics in the various fields of human endeavor.

(c) "Dissemination of knowledge of the parallel cultural development of mathematics with art, literature and science.

(d) "A wider dissemination of certain foundational ideas of mathematics earlier in the student's training.

(e) "The use of favorable propaganda based on fact and on the opinions of authorities and others who have recognized the values of mathematics as a cultural as well as a utilitarian subject."

Each one of the suggestions is elaborated in some detail.

7. Forsyth, A. R. "Applied Mathematics in School Training: Some General Considerations." *The Mathematical Gazette*. 21: 8–24. February, 1937.

A clear analysis of the roles that pure and applied mathematics should play in the curricula designed for general and technical education. In conclusion the author pleads that "whether at school or beyond school, the beginnings of the subject should eschew the veiled opportunities for manipulative exercises in subjects which belong to pure mathematics, and that these beginnings should be connected, in fact and in spirit, with the measurable phenomena of the universe."

8. Ginsburg, Abraham M. "The Sine Function in Nature." *High Points*. Vol. 19, No. 9, pp. 69–70. November, 1937.

The writer believes that the sine function has application in the physical world, compared with which the solution of triangles are relatively trivial. He mentions simple experiments from sound, mechanics, and electricity which, he believes, belong just as properly in the trigonometry class as in the physics room. A crude but instructive device is described for drawing a part of a sine curve on paper or on the blackboard.

9. Gorman, Mary F. "A New Approach to Numerical Trigonometry." *High Points*. Vol. 19, No. 9, pp. 59–62. November, 1937.

The students in the second term of ninth year algebra courses do not grasp completely the unit on trigonometry because of insufficient preparation in the geometric concepts used. To remedy that defect it was decided to lay greater stress on geometry, especially on geometrical construction work, by beginning in the first term and spreading the unit over a longer period of time, introducing the necessary geometric ideas much as is done in the beginning of the geometry course. The method and contents are illustrated by detailed plans of the first nine lessons in the first term, and by some of the supplementary examples in the second term.

10. Leadbeater, J. G. "Indirect Proof in Geometry: A Defence." *The Mathematical Gazette*. 21: 25–32. February, 1937.

The author believes that the disrepute into which the indirect method has fallen "is due solely to bewildering and illogical presentation often given by writers of textbooks. To banish once and for all what we have called the only inherent difficulty, they have merely to impress upon the student that there is but a *seeming illustration*, and that it comes about by a distortion of the figure, *permissible* because it is made necessary by assuming the opposite of what is to be proved. They do not do this. Indeed, it almost seems that they have conspired to be dumb on the subject of distortion, and it is interesting to note that their chief sin of commission is that they themselves indulge in a distortion which is *not permissible*. We may possibly achieve logical proof by assuming the opposite of what we wish to prove; but argument from a figure made not according to specifications must somewhere involve the logical fallacy of an ambiguous term."

The above claims are supported by the quotation and analysis of the indirect proofs found in many British textbooks, especially in connection with the theorem "if a pair of opposite angles of a quadrilateral are supplementary, its vertices are concyclic."

This department cannot afford as much space as would be necessary for a detailed refutation of many of the assertions made in the article. In the issue of *The Mathematical Gazette* for October 1937, a reply may be found by H. G. Forder to some of criticisms made in the article under review.

11. McCormick, Thomas H. "Intelligence and Geometry, with Suggestions." *High Points*. Vol. 19, No. 2, February, 1937, pp. 10–14.

The writer attempts "to establish the fact that there is a definite causal relationship between I.Q. and success in Plane Geometry as indicated by the Regents examinations marks." Two interesting case studies are reported and important implications are drawn. The author concludes that "if a careful division into these three classes—under 99 I.Q. with a special course, 99–112 I.Q. with three terms in which

to do two terms' work, and 113-up I.Q. with regular College Entrance work in regular time—were allowed, there would be a great improvement in the morale and the accomplishments of our Senior High Schools."

12. Read, Cecil B. "Mathematical Magic." *School Science and Mathematics*. 37: 919. November, 1937.

A description of a few interesting mathematical tricks of the "parlor" variety.

13. Sanders, S. T. "Herbert Ellsworth Slaught." *National Mathematics Magazine*. 12: 2–4. October, 1937.

An appreciative but unsentimental necrology of an important figure in the teaching of mathematics in the United States, written by one who knew him personally for more than a quarter of a century. A full page photograph is included.

14. Shouse, J. B. "The Difficulty of the Concrete." *School Science and Mathematics*. 37: 937–945. November, 1935.

A strong protest against the common notion that all the difficulties found in number work are due to abstractness of number. After quoting and analyzing some of the relevant literature, the author concludes that "The problem in arithmetic comes to be seen, then, not primarily as the development of the understanding of abstract number through a succession of concrete experiences, but more largely as the development of ability to apply number knowledge to successive concrete situations."

15. Sweedler, Esther D. "The Mathematics Club at Curtis High School." *High Points*, Vol. 19, No. 2. February, 1937. Pp. 62–64.

A fascinating account of the mathematics that can be learned and of the fun that can be had in handling surveying instruments under the guidance of an enthusiastic teacher. The author concludes with the hope that "The time will soon come when work with field instruments will be an integral part of the high school course in trigonometry approved by the Board of Regents."

The Eleventh Yearbook of the

National Council of Teachers of Mathematics

on

The Place of Mathematics in Modern Education

This yearbook on a timely topic is being issued at an opportune time when a great deal of comment is being made by educators and many others with respect to the importance of mathematics in the schools. The various chapters are written by authorities in the field and should therefore be of interest to teachers of mathematics and others interested in mathematical education.

CONTENTS BY CHAPTERS

Criticisms of Mathematics, Pro and Con. W. D. Reeve.
Reorganization of Mathematics. Wm. Betz.
The Meaning of Mathematics. E. T. Bell.
The Contribution of Mathematics to Civilization. David Eugene Smith.
The Contribution of Mathematics to Education. Sir Cyril Ashford.
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NEWS NOTES

The last meeting of the Bay Section of the California Mathematics Teachers' Association was held on Thursday, October 14, 1937, at the University High School in Oakland. The program was as follows:

"The Relation of Mathematics to Art with Practical Applications" (Dynamic Symmetry): (Illustrated with slides and other visual material). Conducted by Jean Tuttle, Roosevelt High School, Oakland, and Emma Hesse, University High School, Oakland.

"Applications to the Field of Art." Ethel Abeel, Assistant Professor, California School of Arts and Crafts, Oakland.

Discussion: Bernice Cochran, Advisory Chairman, Fremont High School, Oakland; Edith L. Mossman, East Chairman, Garfield Junior High School, Berkeley; Minnie P. Mayme, East Secretary, Willard Junior High School, Berkeley; Adeline Scandrett, West Chairman, Mission High School, San Francisco; Barbara Acheson, West Secretary, Mission High School, San Francisco.

At the First meeting of the Men's Mathematics Club of Chicago on October 15, 1937 Dr. J. M. Kinney of Woodrow Wilson Junior College spoke of the work of Professor H. E. Slaught who passed away last spring.

M. D. Thompson of the Roosevelt High School also spoke on "Mathematics for the Slow Student."

J. S. GEORGES, *Secretary*

At their second meeting Mr. C. E. Jenkins presented a short report on a questionnaire sent out by him to teachers of non-mathematical subjects relative to the kind and amount of mathematics needed in their classes. Professor H. W. Bailey of the University of Illinois also spoke and Mr. Nyberg of Hyde Park High School reported on the work of the Joint Commission of the National Council of Teachers of Mathematics and The Mathematical Association of America.

Associate Professors of Mathematics David V. Widder and Marshall H. Stone of Harvard University have been promoted to full professorships.

The Mathematics Section of the Seventieth Meeting of the Maryland State Teachers Association was held in the Administration Building in Baltimore on October 29th, 1937 with Miss Agnes Herbert presiding. Professor W. S. Schlauch spoke on "Mathematics Useful in Life Planning."

According to a recent issue of the *New York Times:*

"A collection of rare mathematical devices, assembled from all parts of the world in the last thirty-four years by David Eugene Smith, Professor Emeritus of Mathematics at Columbia University, has been presented by him to the university.

"Included in the collection, which was designed to show the development of mathematical science in the last 2,000 years, were many curiosities such as magicians' compasses, tally sticks, portable Chinese sundials no larger than a man's watch, and compasses dating from Roman times.

"About 300 articles were presented to the university by Professor Smith. Among these was a group of fifty sundials made of bronze, silver, wood or ivory, in designs characteristic of the countries in which they were used.

"A portable dial with compass designed for early travelers was identified as the work of Langlois, Parisian instrument maker of the eighteenth century. This device has a universal horizontal dial so that it could be used in any part of the world.

"From Bavaria there was a cubical dial with five faces so that it could be used without a compass. Another model was of Venetian workmanship, probably seventeenth century. Other dials were traced through their numerals or design to Arab-Hindu, Tyrolese, Bohemian, Austrian or Oriental origin.

"Rare astronomical devices used by early navigators, surveying and measuring instruments, tools for calculating and drawing, and old English yardsticks were in the collection.

"Counting devices of the simple 'beads on a string' type were shown in wide variety, from a child's abacus with blue and white beads strung on wire to a huge Arabian counting frame.

"A small bag of matched white bones, used for computing in Korea, modern Buddhist number beads and twentieth century 'lightning multiplicators' were shown, as were German perpetual calendars of the eighteenth century made with two disks moving on a third and capable of showing length of days, nights and months, hours of sunrise and sunset, and days of the week and month."

A joint meeting of the Range Mathematics and Science Clubs was held in Gilbert, Minnesota on Thursday, April 29, at 6:15 P.M. with Mr. O. Schey, Instructor of Mathematics, and Mr. Edward Hribar, Instructor of Science, both of the Gilbert Senior High School in charge of the meeting.

Songs—by the four pages of the Gilbert Senior High School: Rosalie Zgone, Elsie Berquist, Robert Fulton, Alfred Ryan.

Address of Welcome—Superintendent A. R. Holst, Gilbert.

Response—H. Savre, Chairman Science Club.

Violin Solo—Italo Frajola, accompanied by Mable Fred.

Address—"Trends and Possibilities in Curriculum Reorganization" by Dr. Harl R. Douglass, Professor, Secondary Education, University of Minnesota.

SECTIONAL MEETINGS

Mathematics

Report on the Chicago Meeting of the National Council of Teachers of Mathematics—by H. G. Tiedeman, Chairman, Range Mathematics Club.

Panel Discussion of the Revision of the Curriculum in Mathematics, led by Dr. A. D. Gillette, Eveleth.

Science

Round Table Discussion—Teacher, What is Your Answer? Led by H. Savre.

N. E. M. E. A. program Discussion, led by Mr. Edward Hribar, Science Instructor, Gilbert Senior High School.

During a business meeting of the Mathematics Club, the following officers were elected:

President, Mr. H. G. Tiedeman, Mountain Iron Sr. High School.

Vice-President, Miss Esther Gunderson, Hibbing Sr. High School.

Secretary and Treasurer, Miss Lois Pollard, Eveleth Jr. College.

H. G. TIEDEMAN

At the first dinner meeting of Section 19 (Mathematics) of the New York Society for the Experimental Study of Education Dr. Aaron Bakst of Teachers College spoke on "Approximate Computation." At the second dinner meeting in December a panel discussion was held on the topic "Whither Geometry in the Secondary School." Members of the panel were Professors Ralph Beatley of Harvard and M. L. Hartung of Ohio State University, Mr. Wm. Betz of Rochester, N. Y. and Mr. Gordon Mirick of the Lincoln School, New York City.

The twelfth annual conference of Teachers of Mathematics was held at Iowa City on October 15 and 16, 1937.

FRIDAY MORNING, OCTOBER 15, 1937

North Room, Old Capitol

E. W. CHITTENDEN, *presiding*

10:00 A.M. Address: How to Teach for Retention in Algebra," Elsie Parker Johnson, Oak Park High School, Oak Park, Illinois.

10:45 A.M. Address: "Certain Curriculum Hints from Those Working in Business and Industry," Theo E. Donnelly, West Division High School, Milwaukee, Wisconsin.
Discussion

FRIDAY AFTERNOON, OCTOBER 15, 1937

North Room, Old Capitol

J. F. REILLY, *presiding*

1:30 P.M. Address: "The Significance of a Certain Plane Geometry Theorem," Roscoe Woods, University of Iowa.

2:00 P.M. Address: "Experiments with the Contract System Both in Homogeneous and Mixed Classes," Theo E. Donnelly.

3:30 P.M. You are invited to a tea and exhibit of teaching aids, Room 105, University High School.

FRIDAY EVENING, OCTOBER 15, 1937

Iowa Memorial Union

6:00 P.M. Conference Dinner
H. L. RIETZ, *presiding*
Informal Discussion of Effects of Recent Changes in College Entrance Requirements

SATURDAY MORNING, OCTOBER 16, 1937

North Room, Old Capitol

L. E. WARD, *presiding*

9:30 A.M. Address: "Neglected Mathematics in the Teaching of Arithmetic," G. H. Jamison, Northeast Missouri State Teachers College, Kirksville, Missouri.

10:00 A.M. Address "Prognosis of Mathematical Ability in High School," Harry A. Greene, University of Iowa.

10:30 A.M. Address: "The Teaching of Ratio and Proportion," Elsie Parker Johnson.
Discussion

The Buffalo section of teachers of Mathematics held a meeting in Buffalo on Friday Oct. 29th with Principal W. S. Cofran of the Jackson School of Batavia presiding.

FRIDAY MORNING PROGRAM

9:30 Address: "Unifying Themes For Senior High School Mathematics," Dr. M. L. Hartung, University of Minnesota.

10:30 Address: "The Present and Future Status of Mathematics," F. Eugene Seymour, Supervisor of Mathematics State Education Department.

11:15 Discussion of the above addresses Leader—Mr. Seymour.

FRIDAY AFTERNOON PROGRAM

2:00 Business Meeting.

2:15 Address: "Activity Projects in Mathematics," Prof. Carl N. Shuster, State Teachers College, Trenton, New Jersey.

There was an exhibit of instruments and useful models for mathematics teaching.

The Mathematics section of the Maine Teachers' Association held its Annual Meeting at the High School in Portland on Friday Oct. 29, 1937.

Chairman—Miss Dorothy M. Culley, Dover-Foxcroft.

Vice Chairman—Prin. Kenneth W. Bragdon, Cherryfield.

Secretary—Mr. Roy U. Sinclair, Pittsfield.

"A Course in Mathematics not Based on College Requirements," Prof. Virgil S. Mallory, Department of Mathematics, Teachers College, Montclair, N. J.

"Preparatory School Mathematics," Prin. William A. Tracy, Higgins Classical Institute.

"Common Sense in Mathematics," Prin. Harold P. Andrews, Bridgton High School.

"This and That in Teaching Mathematics," Miss Florence A. Day, Brunswick High School.

The Mathematics section of the Middle Tennessee Teachers' Association held its opening meeting on Friday October 29, 1937. The program follows:

Opening Remarks.

Address: "Functional Mathematics," Mr. J. A. Draper, Clarksville, Tennessee.

Address: "Principles of Continuity in Plane Geometry," Mrs. J. B. White, Central High School, Nashville.

Address: "Teaching of the Locus Concept," Mr. R. C. Shasteen, Wartrace, Tennessee.

Address: "Application of Mathematics to Business," Dr. John A. Hyden, Vanderbilt University, Nashville.

Adjourn for General Session.

Mathematics Luncheon, University Club.

Address: "Geometry as a School Subject," Dr. F. L. Wren, Peabody College, Nashville.

Address: "Teacher Training and Improvement of Teachers in Service," Miss Martha Hildebrandt, Chicago, Illinois.

Business Session.

HOUSTON T. KARNES, *President*
MISS MARGARET TUCKER, *Vice-President*
MISS MABEL MEACHAM, *Secretary-Treasurer*

On Saturday, November 6, 1937, Professor J. A. Drushel of New York University presided at a Mathematics panel, which was part of more general conference on "Educational Problems Facing 1938."

The following is a list of the speakers and of the topics upon which they spoke:

"Developing the Course of Study in Arithmetic for the Middle and Upper Grades"—Philip Jaffe, Teacher of Math., Waverly Junior High School, Brooklyn, N. Y.

"Educational Problems Confronting College Departments of Mathematics"—C. W. Munshower, Professor of Mathematics, Colgate University, N. Y.

"Techniques, Old and New, in the Teaching of Mathematics"—R. C. Stearns, Instructor in Mathematics, Hofstra College, N. Y.

"The Teaching of Arithmetic in the Unified Curriculum"—Lillie Geisser, Supervisor, Newwark, N. J.

"Teaching Long Division in the Sixth Grade"—Ruth I. Baldwin, Elementary Supervisor, Westfield, N. J.

"Teaching Intuitive Geometry to Vocational Students"—David Brownman, Vocational Education, Brooklyn, N. Y.

"Stressing Relationships in Teaching Mathematics"—William S. Tobey, Long Branch, N. J.

"Mathematics Problems in the Private School"—Joseph H. Schotland, Hebrew Technical Institute, New York City.

The reading of the papers was followed by an interesting discussion from the floor.

The Mathematic Section (Section 19) of the New York Society for the Experimental Study of Education held the first meeting of the current season on Saturday Evening, Oct. 30, 1937, at the Men's Faculty Club, of Columbia University. The guest of the evening was Dr. Aaron Bakst, of Teachers College, Columbia University, who spoke on the topic "Certain Aspects of the Teaching of Approximate Computation." A spirited discussion followed the lecture.

A Local Mathematics Club

Many teachers think that little can be done in their school community to promote the work of the National Council of Teachers of Mathematics, that they must wait for a State or Sectional Mathematics Association to affiliate with the National Council to carry on any definite line of work that will advance the cause of Mathematics. This is not the case, more can be accomplished in smaller groups that meet frequently, if not regularly, than in a large organization.

The Wichita Mathematics Association is a local association of teachers of Mathematics. It was organized in April, 1935, affiliated with the National Council in October, 1936. Several new members have been added to the National Council since its organization.

The purpose of this association is: to arouse interest in the study and teaching of Mathematics; to help the teachers of Mathematics in this city to become better acquainted, to talk over their teaching problems; to enable the teachers to gain a better understanding of the Mathematics work from the elementary grades through the high school and college courses; to bring outside speakers before our local group as an incentive and inspiration to the teachers to keep abreast of the times in the teaching of Mathematics; to further the interests of our State Mathematics Association and to do our part in helping the National Council of Teachers of Mathematics reach its goal of ten thousand members.

During the year, six meetings were called by the President, Arther J. Hoare.

October—East High School—A tea and program. Guest speaker, Dr. A. M. Harding of the University of Arkansas gave an interesting talk on Astronomy and the Proposed New Calendar.

November—North High School—A musical program, and Report of the various State Round-Table Meetings.

January—Central Intermediate School—Program—Music, A Mathematics Reading. Address: J. E. Stinson, Principal of Allison Intermediate School—Mathematics' Place in the Curriculum.

March—University of Wichita—Tea and program. Address: Supt. L. W. Mayberry, Mathematics in Modern Life.

April—The State Association of Teachers of Mathematics met in Wichita. William Betz—Director of Mathematics, Rochester, New York gave two excellent addresses on the Reorganization of Secondary School Mathematics.

May—Horace Mann Intermediate School. Report of the Annual Meeting of the National Council at Chicago—February 19, 20. Election of officers: President—Arthur J. Hoare, University of Wichita: Vice-President—Roy Taylor Allison Intermediate School; Secretary—Ruth Woodworth, East High School; Treasurer—Edith Webster, Alexander Hamilton Intermediate School; Program Chairman—Edwin A. Beito, University of Wichita; Local Editor—Mary Kelly, East High School.

The association has seventy-five members, a third of the members are also members of the National Council. The membership includes mathematics teachers from the University of Wichita, Friends University, Mount Carmel Academy, North and East High Schools, the six Intermediate Schools, the Parochial Schools, and several persons outside the schools who are interested in mathematics.

The organization of the Wichita Association of Teachers of Mathematics has done more to create good will, interest, and earnest effort in the teaching of mathematics than anything yet tried.

If there is a lack of funds to secure speakers, an entertainment may be given that will be both instructive and entertaining to the students and the public.

Organize a local Mathematics Club for the teachers of Mathematics in your city this year. Vitalize the teaching of Mathematics. Affiliate with the National Council. Secretary Edwin W. Schreiber, Macomb, Illinois, will furnish the necessary blanks for affiliation.

MARY KELLY
Vice-President of National Council

The Women's Mathematics Club of Chicago held its first meeting on Oct. 30, 1937. The program consisted of a special demonstration at the Adler Planetarium by Miss Maud Bennot on the "Annual Journey of the Sun," and a visit to the Permanent-Changing Exhibit of High School Mathematics housed there.

Officers of the club are:

Pres. Dorothy Martin, Chicago Heights High School; Vice-pres. Ida Fogelson, Bowen High School; Sec. Helen White, Englewood High School; Treas. Mary Jane Hartmann, Oak Park High School.

IONIA J. REHM

The Association of Teachers of Mathematics of the Middle States held their annual meeting at Atlantic City, (Haddon Hall) on November 27, 1937.

HARRY C. BARBER, of the English High School, Boston, Mass., spoke on the topic "*Computing with Mathematical Inaccuracy (Approximate Calculation).*"

C. N. SHUSTER, of State Teachers' College, Trenton, N. J., followed with "*Practical Applications for Junior and Senior High School.*"

GORDON R. MIRICK, of Lincoln School of Teachers' College, discussed the much debated question of College Entrance Examinations. His topic—"*What a Progressive School is Doing to Prepare for the Alpha Beta Gamma Examinations,*" and "*Suggested Modifications of Books VII, VIII, and IX of Solid Geometry,*" was of interest to everyone.

The Standing Committee, of which D. E. McCORMICK, Penn Charter School, is Chairman reported JONATHAN T. RORER, *President;* JAMES ADAMS, *Vice-President;* J. BURNETTE HALLOWELL, *Treasurer.*

RUTH WYATT, *Secretary*

A National Council luncheon was held on Wednesday, June 30, 1937, at the Detroit Leland Hotel. This luncheon was a "Group Discussion Luncheon." Following were the topics and leaders:

1. Mary Kelly, Wichita, Kan.: How shall we develop an appreciation of mathematics in the modern school?

2. J. T. Johnson, Normal School, Chicago, Ill.: The ninth grade course in mathematics.

3. Edwin W. Schreiber, State Teachers College, Macomb, Ill.: Mathematics for teachers of mathermatics who teach the masses.

4. W. D. Reeve, Teachers College, Columbia University, New York City: The next step in mathematics education.

5. H. C. Christofferson, Miami University, Oxford, Ohio: Beginning geometry with constructions.

6. Kate Bell, Lewis and Clark High School, Spokane, Wash.: Sources of illustrative material.

7. Eleanor Booher, Oak Park and River Forest Twp. H. S. Oak Park, Ill.: What shall we teach during the first six weeks in geometry?

8. L. S. Johnston, Professor of Mathematics, College of Engineering, University of Detroit, Detroit, Mich.: Supplementary projects for the bright student in high school mathematics.

9. Nellie Loss, Flint, Mich.: Procedures in teaching dull pupils.

10. Brown Miller, Fairmont Junior High school training school, Cleveland, Ohio: Extra-curricular activities in Junior High School mathematics.

11. Florence Brooks Miller, Shaker Heights Jr. H. S., Cleveland, Ohio: The contribution mathematics makes to other junior high school subjects and how these may be used to enrich mathematics.

12. Frances A. Mullen, Fenger High School Chicago, Ill.: Specific sources of supplementary material which may be put into the hands of ninth year algebra students.

13. Raleigh Schorling, University of Michigan, Ann Arbor, Mich.: Presenting the value of mathematics training in an educational guidance program of high school students.

14. Ben. A. Sueltz, State Normal School, Cortland, N. Y.: Mathematics rather than arithmetic in the elementary school.

15. Helen Taylor, Department of Tests and Measurements, University of Illinois. Urbana, Ill.: Building a good remedial program based on diagnostic testing.

16. C. L. Thiele, Supervisor of Exact Sciences, Detroit, Mich.: Mental Arithmetic.

The First Annual Meeting of the Nebraska Section of the National council of Teachers of Mathematics was held in Lincoln on May 8, 1937.

Program Theme—"The Present Crisis in High School Mathematics and What High School Teachers of Mathematics May Do to Meet It."

Part I

Highlights of The National Meeting at Chicago

A man's view—Prof. Carl R. Thomas, Teachers College, Chadron.

A Woman's view—Miss Amanda Anderson, Central High, Omaha.

Part II

High School Mathematics from five Viewpoints

The Classroom Teacher—Miss Jennie Walker Norfolk High.

Social Science—Mrs. Grace Hyatt, Lincoln High.

Administration—Prin. H. C. Mardis, Lincoln High.

Sciences—Prof. J. E. Brock, Teachers College Wayne.

Pure Mathematics—Dr. W. C. Brenke, Nebraska University.

Part III

Luncheon

The Ideal Mathematics Teacher—Dr. A. R. Congdon, Teachers College, University of Nebraska.

OFFICERS EXECUTIVE BOARD: Carl R. Thomas, Chadron, Chairman; Eva Phalen, Kearney, Vice-Chairman; Lloyd V. Cross, Norfolk, Secretary; Ellen V. Anderson, Lincoln, Treasurer; Amanda E. Anderson, Omaha, Editor; Inez Wilson, Curtis, Dr. A. R. Congdon, Lincoln, N.C.T.M. Representative.

Mathematics section meetings were held in each of the six districts as a part of the annual sessions of the Nebraska State Teachers Association Oct. 28–29 and the following programs were presented.

District No. 1 at Lincoln

President: A. Gaylord Wilson, Pawnee City.
Secretary: Margaret Morton, Superior.
Theme: "Humanized Mathematics."
Music.
"Teaching Human Attitudes and Appreciations in Algebra," Prin. Duane Perry, Louisville
Discussion, led by Mr. Perry.
"A Human Touch in the Teaching of Mathematics," Acting President W. H. Roselius, Hebron College and Academy.
Election of Officers.
Talking Motion Picture, "The Play of Imagination in Geometry", David E. Smith, Columbia University.

District No. 2 at Omaha

President: Mary Doremus, Fremont.
Secretary: Earl Bragg, Omaha.
Friday, 1:30 p. m.
"The Improvement of Instruction in Secondary Mathematics," Arthur E. Mallory, Professor of Mathematics, Colorado State College of Education, Greeley.
Discussion.

District No. 3 at Norfolk

Chairman: Floyd Schelby, Madison.
Business Meeting: "The Effect of Remedial Work in Reading Comprehension upon Algebraic Achievement," Mildred I. Clark, Wayne.
Address, Dr. A. R. Congdon, University of Nebraska.

District No. 4 at Grand Island

President: Clarence Lindahl, Paxton.
Secretary: Walter Gall, Cozad.
"Report of Nebraska Council of Mathematics Teachers," Eva Phalen, Kearney.
Round Table Discussion led by Professor M. S. Pate. Kearney State Teachers College.

District No. 5 at Holdrege
Science and Mathematics

"Edward Schaefer, Holbrook, presiding.
"A Practical Science Curriculum for Nebraska High Schools," Dr. Harold Wise, University of Nebraska, Lincoln.

District No. 6 at Scottsbluff

President: Lena M. Meyers, Kimball.
Secretary: Mrs. Nellie Barron, Scottsbluff
"Evaluation of Teaching Materials," Dr. John Guy Fowlkes, University of Wisconsin, Madison.
"Special Methods in Teaching Algebra," Lillian Burns, Alliance.
"The Nebraska Mathematics Organization," Prof. Carl R. Thomas, Teachers College, Chadron.

The above summary was prepared by Prof. Arthur L. Hill, of the mathematics department of Peru State Teachers College.

ARTHUR L. HILL

On Saturday morning, November 20, 1937 there was held at Washington Irving High School, a meeting to reorganize the Mathematics Association of New York City.

Mr. Henry H. Shanholt, President, spoke about the past work of the Association and the need for its revival. Mr. Joseph Orleans addressed the meeting on behalf of the Mathematics Chairmen's Association. A letter from Miss Margaret C. Byrne, on behalf of the Standing Committee on Mathematics, was read.

The election of the following temporary officers followed:

President—Mr. H. Sitomer, New Utrecht H. S.

Vice-President—Miss Etta Greenberg, Washington Irving H. S.

Rec. Secretary—Mrs. Henrietta Mazen, Theo. Roosevelt H. S.

Fin. Secretary—Mr. J. Deutsch, Thomas Jefferson H. S.

In addition, an Executive Board of twenty-three was unanimously chosen.

The Association launched a program of activity which will include a study of the pressing problems facing teachers of mathematics in the Junior, Senior, Technical and Vocational High Schools of N. Y. C.

HENRIETTA MAZEN
Temporary Recording Secretary

Dr. Herman von Baravalle formerly of the Waldorf Schule in Stuttgart, Germany, but now of the Edgewood School in Greenwich, Connecticut, will be the guest and speaker at the third dinner meeting of Section 19 (Mathematics) of the New York Society for the Experimental Study of Education. He will give an illustrated lecture on the topic "The Geometry of the Five-Angle."

ON TO Atlantic City! See the Program of the Nineteenth Annual Meeting of the National Council of Teachers of Mathematics on the following pages of this issue.

Program of the Nineteenth Annual Meeting

Of The National Council of Teachers of Mathematics, Traymore Hotel, Atlantic City, New Jersey, February 25-26, 1938

THURSDAY, FEBRUARY 24, 8:00 P.M.

Informal Meeting of the Board, Club Room, 10th Floor

FRIDAY, FEBRUARY 25, 10:30 A.M.

Board Meeting, Club Room, 10th Floor

FRIDAY, FEBRUARY 25, 2:00 P.M.

I. Teacher Training Section, Rose Room Mezzanine Floor

Presiding: Vera Sanford

1. What Professional Training is Needed for Teaching Arithmetic in the Elementary Schools?
   Clifford B. Upton, Teachers College, Columbia University
2. The Teacher's Professional Study of Subject Matter in Mathematics
   William C. Bagley, Teachers College, Columbia University
3. Training in Mathematics Desirable for Teachers in the Elementary Schools
   Herman M. Cooper, Assistant Commissioner of Education in Charge of Teacher Training and Certification, State Department of Education, Albany, New York
4. What Academic Training in Mathematics Should Be Required of all Students in the 3 and 4 Year Normal School?
   C. N. Shuster, State Teachers College, Trenton, New Jersey
   E. H. Taylor, Eastern Illinois State Teachers College, Charleston
   L. H. Whitcraft, Ball State Teachers College, Muncie, Indiana

II. *Reports on the Status of Mathematics in the United States*, Belvedere Room, 11th Floor

Presiding: Florence Brooks Miller

1. A Brief History of the National Council of Teachers of Mathematics
   Edwin W. Schreiber, Macomb, Illinois
2. Activities of the National Council of Teachers of Mathematics in Tennessee
   F. L. Wren, Nashville, Tennessee
3. Glimpses of Plans and Procedures from Secondary Mathematics Teaching in California
   Edith L. Mossman, Berkeley, California
4. The Status of Mathematics as a Requirement for University Entrance in Nebraska
   A. R. Congdon, Lincoln, Nebraska
5. The Mathematics Clearing House in Wisconsin
   Mary A. Potter, Racine, Wisconsin
6. A Report from Connecticut
   Dorothy S. Wheeler, Hartford, Connecticut
7. Certain Curriculum Changes to Meet the New Requirements of the College Entrance Board
   H. Gray Funkhouser, Exeter, New Hampshire
8. Organization and Work of Committees of Men's Mathematics Club of Chicago and Vicinity
   J. T. Johnson, Chicago, Illinois
9. A Proposed National Project in Mathematics
   William Betz, Rochester, New York

FRIDAY, FEBRUARY 25, 4:00 P.M.

Meeting of State Representatives and Delegates to the National Council of Teachers of Mathematics, Belvedere Room, 11th Floor

FRIDAY, FEBRUARY 25, 8:00 P.M.

General Meeting

1. Address of Welcome—Speaker to be announced.
2. Report of the Work of the Joint Commission of the Mathematical Association of America, and the National Council of Teachers of Mathematics on the Place of Mathematics in the Secondary Schools
   K. P. Williams, Bloomington, Indiana, Chairman; A. A. Bennett, Providence, Rhode Island; M. L. Hartung, Columbus, Ohio; Ruth Lane, Iowa City, Iowa; C. A. Hutchinson, Boulder, Colorado; Discussion

SATURDAY, FEBRUARY 26, 1938, 9:00 A.M.

Arithmetic Program, Stratosphere Room, 11th Floor

Presiding: H. C. Christofferson

1. Deferred Arithmetic

Leo J. Bruckner, University of Minnesota, Minneapolis, Minnesota
G. T. Buswell, University of Chicago, Chicago, Illinois

2. Work of the Arithmetic Committee, Progress to Date
R. L. Morton, Ohio University, Athens, Ohio, Chairman
3. Discussion
Leader, J. T. Johnson, Chicago, Illinois

High School Section, Submarine Grill

Presiding: Martha Hildebrandt

1. The Nature and Place of Objectives in Teaching Geometry.—E. R. Breslich, University of Chicago, Chicago, Illinois
2. Geometries.—Mayme I. Logsdon, University of Chicago, Chicago, Illinois
3. Applying Geometric Methods of Thinking to Life Situations.—Elizabeth L. Hall, Benjamin Franklin High School, Rochester, N. Y.
4. An Excursion into the Realm of the Imaginaries.—Luise Lange, Woodrow Wilson Junior College, Chicago, Illinois

SATURDAY, FEBRUARY 26, 11:45 A.M.

Discussion Luncheon

For Discussion Luncheon reservations send your first, second, and third choice of topic and a check for $1.40 (including tip) to E. H. Hildebrandt, State Teachers College, Upper Montclair, New Jersey.

Discussion Leaders and Topics

1. William Betz, Rochester, New York
Remedial Arithmetic Work for Ninth Grade Classes
2. E. R. Breslich, Chicago, Illinois
Testing in Mathematics
3. H. C. Christofferson, Oxford, Ohio
Geometry, A Way of Thinking
4. J. W. Colliton, Trenton, New Jersey
Mathematics for Gifted Pupils
5. Harold Fawcett, Columbus, Ohio
The Nature of Proof
6. H. Gray Funkhouser, Exeter, New Hampshire
Use and Procurement of Supplementary Materials
7. L. D. Haerrter
Desirable Qualities which the Study of Mathematics can Develop
8. M. L. Hartung, Columbus, Ohio
What are Some Promising Methods of Teaching for Transfer
9. G. H. Jamison, Kirksville, Missouri
The Development of Curiosity as an Incentive to Further Mathematical Study
10. Amanda Loughren, Elizabeth, New Jersey
Need for the Continuity of Instruction in Arithmetic During the Secondary-school Period
11. Joy Mahachek, Indiana, Pennsylvania
How has the Recent Tendency in Grade Placement of Topics in the Elementary School Affected Mathematics in the Elementary School? in the Secondary School?
12. Virgil S. Mallory, Montclair, New Jersey
Calculus and Analytic Geometry in High School and its Implications
13. Joseph P. McCormack, New York City
Making Ninth Year Mathematics Transfer to Life
14. Frank M. Morgan, Hanover, New Hampshire
The teaching of Plane Geometry
15. Mary Potter, Racine, Wisconsin
Practical Uses of Mathematics We Have Discovered in Our Town
16. Carl N. Shuster, Trenton, New Jersey
Approximate Computation in High School Mathematics
17. Joseph Seidlin, Alfred, New York
What is Wrong with the Way I Teach?
18. Rolland R. Smith, Springfield, Massachusetts
Preparing for the Alpha, Beta, Gamma Examinations
19. C. Newton Stokes, Philadelphia, Pennsylvania
Methods of Teaching for Retention
20. Ben A. Sueltz, Cortland, New York
Algebraic Thinking in the Elementary School
21. Ethel Sutherland, New York City
The Grade Placement of Topics of Arithmetic in the Elementary School
22. Clifford B. Upton, New York City
Modern Business Arithmetic as a Cultural Subject in Grades 7 to 9
23. Edith Woolsey, Minneapolis, Minnesota
How Much Algebra Shall We Teach in a Ninth Grade Course on Everyday Mathematics?
24. F. L. Wren, Nashville, Tennessee
The Concept of Dependence in the Teaching of Geometry
25. Joseph Jablonower, New York City

Guide Lines in the Choice of Curriculum Material in Mathematics

26. Ruth Lane, Iowa City, Iowa
Pupil Selection of Graded Originals in Geometry

27. Walter W. Hart, Winter Haven, Fla.
General Mathematics for Ninth Grade—not College Preparatory

28. L. H. Whitcraft, Muncie, Indiana
Topic to be announced

SATURDAY, FEBRUARY 26, 2:15 P.M.

I. *Junior High School Program*, Stratosphere Room, 8th floor

Presiding: Virgil S. Mallory

1. Glimpses from the Baltimore Classrooms
Nanette L. Roche, Supervisor of Mathematics, Baltimore, Md.

2. A Unit in Mathematical Geography Introducing Rectangular and Polar Coordinates at the Junior High School Level
E. H. Hildebrandt, State Teachers College, Montclair, New Jersey

3. The Unit of Statistics in Ninth Grade Mathematics—Goals, Methods, Difficulties
Helen Walker, Teachers College, Columbia University

II. *Senior High School Program*, Submarine Grill

Presiding: Martha Hildebrandt

1. A Means of Improving the Relation between Secondary-school and College Mathematics
Ralph Beatley, Harvard University, Cambridge, Mass.

2. The New Attainment Tests of the College Entrance Examination Board and a More Significant Preparation in Mathematics
J. R. Kline, University of Pennsylvania, Philadelphia

3. Discussion

SATURDAY, FEBRUARY 26, 3:45 P.M.

Business Meeting of the National Council of Teachers of Mathematics Stratosphere Room, 8th Floor

Annual Banquet, 6:30 P.M. $2.20 per plate (including tip)

Address: Take-and-Put Algebra
William R. Ransom, Tufts College, Massachusetts

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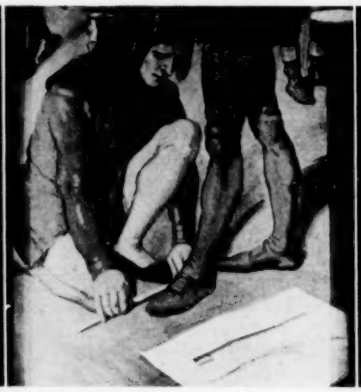
—Cubit—
First known measurement.
About 20 inches.
Length of forearm from point of elbow to end of the middle finger.

—Palm—
Width across an open hand at base of fingers.
About three inches.

—Digit—
Breadth of finger.
From .72 to .75 inch.

—Yard—
King Henry I decreed.
The distance from the point of his nose to the end of his thumb was the lawful yard.



—Inch and Foot—
Three barley corns taken from center of ear placed end to end equals one inch. (Edward II 1324)
A foot ranged from 9¾ inch to 19 inches.

—Rod—
16th Century.
The lawful rod was the length of t
left foot of 16 men lined up as th
left church on Sunday morning.

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